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Dear readers,

in your hands you are holding the booklet of the 27th volume of Náboj Physics. The booklet contains all the physics problems you could have encountered during the competition this year, as well as the solutions to them, from which you can learn a lot. If you have any difficulty understanding any of them, do not hesitate to contact us; we will gladly clarify everything.

This booklet would not exist without the enormous effort of many people who participated in organising Náboj Physics. Most of us are students of Faculty of Mathematics, Physics and Informatics of Comenius University in Bratislava, and some of us also actively participate in organising the Physics Correspondence Seminar (FKS).

Náboj Physics continues with its international tradition as well in the year 2024. For the international cooperation, we would like to thank to the local organisers: Katarína Nedeľková (Bratislava), Marián Kireš (Košice), Jakub Kliment (Praga), Lenka Plachtová (Ostrava), Ágnes Kis-Tóth (Budapest), Urszula Goławska (Gdansk), Andrzej Karbowski (Torún), Mirela Kaczmarek (Breslavia), José Francisco Romero García (Madrid) and Dmytro Rzhemovskyi (Viena).The results of this international clash can be found on our web site.

In the name of the entire team of organisers, we believe that you enjoyed Náboj Physics in 2024, and we hope that we see each other at Náboj next year. Either as competitors or organisers.

Jaroslav Valovčan
Chief organiser

The results, the archive and other information can be found on the web site `https://physics.naboj.org/`.

Problemas

1 ¡95 % de la gente no puede resolver este problema! ¿Y tú?

🍌 ÷ (🍒 × 🍒) = 🍇

🍉 × (🍌 × 🍌) × 🍇 = 🍋

🍋 ÷ 🍌 = 2,7 kJ

🍋 ÷ ((🍌 × 🍌) × (🍌 × 🍌)) = 12,5 Pa

🍉 × (🍇 × (🍌 ÷ 🍒)) = 675 W

(🍉 × 🍌) ÷ (🍒 × 🍒) = 450 N

$$\frac{\dfrac{(\text{🍉}+\text{🍉})\times(\text{🍉}+\text{🍉})}{(\text{🍉}+\text{🍉})+(\text{🍉}+\text{🍉})}\,(\text{🍌}+\text{🍌})^{\frac{(\text{🍌}+\text{🍌})}{\text{🍌}}}}{((\text{🍒}+\text{🍒})\times\text{🍒}) - \dfrac{\text{🍋}}{(\text{🍋}+\text{🍋})}\times\dfrac{\text{🍌}}{\text{🍇}}} = \text{? Wh}$$

2 Thomas estaba viendo una carrera de coches. Esta vez una carrera americana, que implica girar a la izquierda durante tres horas. Y como nunca ahorró suficiente dinero para viajar a América y verla en directo, llamó a Pato y Josef y les pidió que corrieran para él en la rotonda local.

Los chicos llegaron en sus cacharros, pararon uno al lado del otro y Thomas inició la carrera. Ambos conducen a una velocidad de vértigo constante de 18 km/h. Pato se queda en el carril interior, un círculo con una circunferencia de 100 m, mientras que Josef conduce por el carril exterior, cuya circunferencia es de 120 m ¿Cuántas vueltas habrá dado Pato a la rotonda para cuando adelante a Josef por primera vez?

No tengas en cuenta el tiempo que necesitan los chicos para alcanzar la velocidad de vértigo.

3 Martin observa a menudo lluvias de estrellas. Sus preparativos para las observaciones nocturnas incluyen, entre otras cosas, preparar café en un termo (siete cucharaditas de café soluble y nueve cucharaditas de azúcar, mezclado, no agitado). El recipiente interior del termo es un cilindro con una altura de 18 cm y un radio en la base de 4 cm. Las paredes del contenedor interior tienen un grosor de 0.5 mm y están hechas de aluminio con una densidad de 2.7 g/cm^3 y un calor específico de 0.9 $J/(g \cdot K)$. Martin ha llenado completamente el recipiente interior con café a una temperatura de 95 °C.

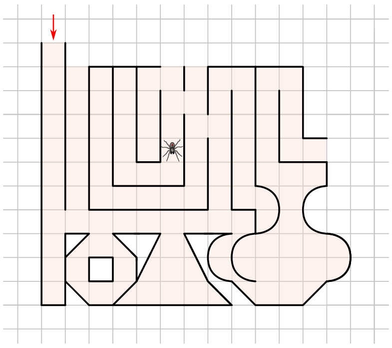
¿Cuánto se enfriará el café cuando alcance el equilibrio con el matraz interior? El matraz interior está separado del exterior por un vacío, por lo que se puede suponer que está perfectamente aislado térmicamente. La capacidad calorífica específica del café es igual a la del agua. La temperatura original del matraz era de 20 °C.

4 Mateo por fin ha subido al avión. Ahora está a punto de despegar. Como viajero experimentado le son completamente indiferentes los retrasos, los niños gritones y los desagradables bocadillos secos, pero como es físico sigue interesado en los datos del vuelo. Así que saca su teléfono, que le muestra que el avión está acelerando a 2 m/s^2. Sin embargo, la velocidad del avión suele expresarse en km/h.

¿Cuál es la aceleración del avión en las unidades km/h^2?

5 Justine y Martin reciben un regalo de boda: una creación de cristal hecha por vidrieros profesionales en Venecia. Sin embargo, pronto una araña se introdujo en el interior del regalo. No parecía muy estética allí, así que decidieron echarla.

¿Cuánta agua tienen que verter en el tubo de más a la izquierda para que el cuadrado con la araña se llene por completo? Considera que un cuadrado corresponde a un volumen de 1 l y las dimensiones son lo suficientemente pequeñas como para que pueda **despreciarse** la compresión del aire.



6 George midió en el globo terráqueo que la distancia de Río a Hong Kong es de 17 700 km y la distancia de Río a Tokio es de 18 600 km. ¿Cuál es la distancia máxima posible entre Tokio y Hong Kong?

La Tierra es una bola con una circunferencia de 40 000 km. *No supongas nada sobre la ubicación en el mundo real de los lugares mencionados.*

7 Dos pilotos experimentados de Trackmania, Thomas y Matthew, compiten en una larga recta de 300 m. El coche de Thomas empieza a acelerar desde la línea de salida con una aceleración constante de 8 m/s^2. El coche de Matthew tiene problemas de encendido y sólo arranca un segundo después, pero con una aceleración de 9 m/s^2.

¿Quién cruzará primero la línea de meta y por cuántos segundos ganará?

8 Dé el resultado de la suma de los números asociados a las afirmaciones que sean ciertas:

1	El agua está fluyendo por una tubería a través de un punto estrecho. La velocidad del flujo es menor allí.
2	Si ejercemos fuerza sobre la superficie de un líquido, la presión es la misma en todos los puntos de su interior.
4	Un objeto flota en la superficie del agua de un vaso. Por lo tanto, su densidad debe ser menor que la del agua.
8	Un cilindro estrecho de vidrio está medio lleno de mercurio. Al inclinarlo 45°, la presión en el fondo disminuye.
16	Un cubito de hielo flota en la superficie del agua de un vaso a temperatura ambiente. El nivel del agua subirá hasta que se derrita por completo.
32	El agua no puede evaporarse a temperaturas muy inferiores a su punto de ebullición.

64	Dos objetos flotan en la superficie de una masa de líquido. El objeto cuyo volumen sobre la superficie del líquido es mayor es menos denso.

Todos los objetos mencionados se encuentran en condiciones ambientales estándar (campo gravitatorio homogéneo, presión atmosférica, temperatura ambiente, etc.)

9 Las ventanas de nuestra piscina cubierta local están cubiertas por gruesas cortinas. En una de ellas hay un pequeño agujero a través del cual un rayo de luz solar incide en la superficie del agua de la piscina con un ángulo de 45°. Como la luz solar está formada por componentes de colores, en el fondo de la piscina aparece una franja arco iris. ¿Cuál es la longitud de esta franja si el índice de refracción del agua para la luz visible se encuentra en el intervalo de 1.33 – 1.34 y la piscina es 2 m de profundidad.

Asuma que el índice de refracción del aire es 1.

10 Durante los largos y descuidados días de verano, Marc olvidó echar anticongelante en el depósito de líquido limpiaparabrisas de su coche, dejando allí agua sin gastar. Ahora está congelado delante de su coche, maldiciendo y gesticulando salvajemente, y poco a poco se da cuenta de que en el depósito de dos litros hay 1 kg de hielo puro.

Lo llena hasta arriba con líquido limpiaparabrisas de invierno, que se congela a −20 °C. ¿Cuál es la temperatura mínima a la que la mezcla resultante será líquida si la densidad del anticongelante es de 800 kg/m^3 y el punto de congelación es una media ponderada simple basada en la masa de los constituyentes?

11 En la feria local, Fran compró un donut y una manzana enorme. Se comió el donut inmediatamente y guardó la manzana para volver a casa. Al pasar por un pequeño puente, vio su propio reflejo en el agua. Esto le inquietó tanto que hizo que se le cayera la manzana al estanque. Resultó que la densidad de la manzana era dos tercios de la densidad del agua.

Como Fran aún tenía las manos grasientas del donut y no quería provocar un desastre medioambiental, se arrodilló y empezó a masticar la manzana justo por encima del nivel del agua. Sin embargo, el repentino chapoteo también atrajo a un pez hambriento, que empezó a comerse la manzana también por debajo de la superficie.

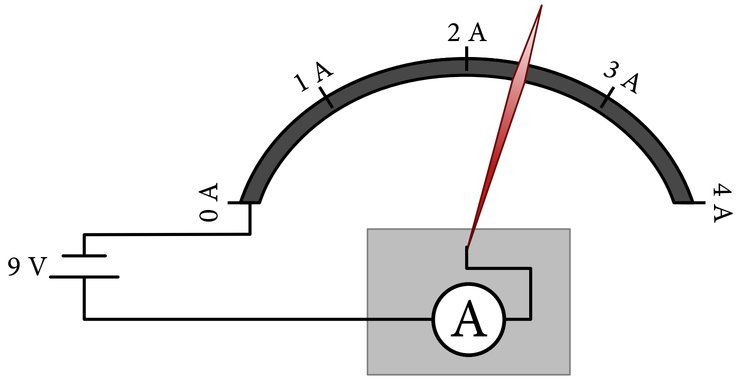
¿Qué parte de la manzana se habrá comido al final Fran? Dada la enorme ventaja en términos de dientes de Fran, come tres veces más rápido que el pez.

12 Jeremy baja la colina a toda velocidad en su bicicleta. A su velocidad actual y en terreno llano, necesitaría una distancia s y un tiempo t para detenerse. ¿Cuál es la inclinación mínima de la colina respecto al plano horizontal que impediría a Jeremy detenerse?

Jeremy sabe frenar correctamente y sus ruedas nunca patinan.

13 Jacob buscaba un multímetro en casa, pero, por desgracia, sólo encontró un extraño amperímetro. Su aguja se mueve a lo largo de una fina tira conductora con una resistencia de longitud de 4 Ω por segmento, tal como se muestra en la imagen. Conectó este milagro de la ingeniería eléctrica moderna a una pila con una tensión de 9 V.

¿Qué corriente indicará el extraño amperímetro?



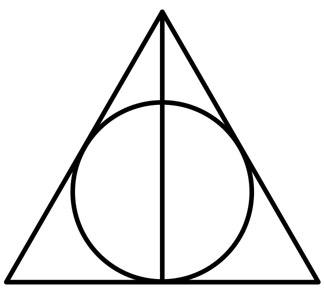
14 Patrick descubrió un extraño disco negro en el armario de su abuelo. Antes de que pudiera servir una hamburguesa de soja en él o utilizarlo en lugar de un frisbee, su abuelo le enseñó que si lo colocaba en el tocadiscos, colocaba la aguja en la ranura en espiral del borde exterior del disco y, por último, pulsaba unos botones, de los altavoces empezarían a salir ominosos crujidos y siseos, junto con algo de música prehistórica.

El silbido y el crujido duran exactamente 20 minutos. ¿Cuál es la longitud de la ranura si el diámetro exterior del disco es 30 cm y la ranura termina a 5 cm de su centro? El misterioso disco giraba en el tocadiscos a una velocidad de 33⅓ revoluciones por minuto.

15 Un saltador se lanza a una piscina desde un trampolín que está 10 m por encima del nivel del agua. Rebota en el trampolín, da una voltereta y aterriza en el agua. ¿Cuánto tiempo pasa el saltador en el aire, si ha pasado exactamente la mitad de la duración del salto por encima del nivel del trampolín?

16 Sarah es una gran fan de Harry Potter. Sacó de su joyero un colgante de las Reliquias de la Muerte y empezó a admirarlo: la Capa de Invisibilidad (el triángulo), la Piedra de la Resurrección (el círculo), la Varita de Saúco (la altura). Y como también es una física curiosa, se preguntó dónde se encuentra el centro de masa de este colgante.

El colgante está formado por un triángulo equilátero de lado a, un círculo inscrito en el triángulo y una de sus alturas. Todos estos componentes tienen la misma densidad lineal. ¿A qué distancia se encuentra el centro de masa del colgante del vértice superior del triángulo?



17 Andrew construyó una catapulta primitiva. Consiste en un tablón sin masa de longitud $2L$ con una bisagra en su centro, que se sujeta a un tosco armazón a una altura L del suelo. Pega un pesado contrapeso de piedra con masa M en un extremo y coloca un diminuto proyectil con masa $m < M$ en un agujero poco profundo en el otro extremo.

El proyectil permanece allí hasta que Andrew tira del pestillo y dispara la catapulta. La pesada piedra desciende, y el proyectil sale del foso, justo cuando el tablón está vertical.

¿A qué distancia del contrapeso impactará el proyectil contra el suelo?

18 La autopista de Bratislava a Praga mide exactamente 400 kilómetros y tiene cuatro carriles en cada sentido. En esos carriles, los coches circulan a velocidades de 80 km/h, 100 km/h, 120 km/h y 160 km/h. Pedro, Pablo y Arturo salieron de Bratislava al mismo tiempo, pero cada uno tomó un carril distinto porque cada coche soporta velocidades diferentes. Cuando llegaron a Praga, contaron sus experiencias:

Peter: «Conduje por el carril más rápido y adelanté a 620 coches por el camino, ¡y de ellos, 200 iban por el segundo carril más rápido!».

Paul: «¡Conduje a una velocidad de 120 kilómetros por hora y adelanté a un total de 220 coches!».

Arthur conducía a una velocidad de 100 km/h, pero no recuerda cuántos coches adelantó. Calcúlalo por él.

Supongamos que los coches salen de Bratislava por cada carril de manera uniforme y que todos recorren la distancia completa.

19 A Matt le gusta montar a caballo con sus físicos amigos. Hoy se han dado cuenta de que si

- la *longitud de un caballo* es de ocho pies;
- un *caballo de fuerza (HP)* es la potencia necesaria para levantar una masa 75 kg hacia arriba a una velocidad de 1 m/s;
- y una *ración de caballo* es 1.7 kg de alimento por 100 kg de peso de un caballo por día;

es posible construir un sistema de medidas en el que se pueda expresar cualquier magnitud mecánica. Por ejemplo, la *aceleración de un caballo* sería la *longitud de un caballo* multiplicada por la *ración de un caballo* al cuadrado.

¿Cuántos kilogramos es *un caballo de peso*, es decir, una gran unidad de masa derivada de estas tres unidades?

20 Cathy y Billy viajan en una escalera mecánica. Tiene una longitud de 120 escalones visibles y se mueve a una velocidad de dos escalones por segundo. Ambos pisan el escalón más bajo. Cathy, como adulta responsable, se queda allí tranquilamente, mientras Billy empieza a correr frenéticamente entre ella y la parte superior de la escalera mecánica, yendo y volviendo, a una velocidad de dos escalones por segundo al subir y de seis escalones por segundo al bajar.

¿Cuántos escalones habrá saltado Billy en total cuando Cathy lo alcance en lo alto de la escalera mecánica y le dé una buena reprimenda?

21 Marc y Sabine vuelan en dos aviones supersónicos, cada uno a una velocidad de Mach 3, a lo largo de dos líneas paralelas separadas 1 km en sentidos opuestos. ¿Cuánto tiempo transcurrirá entre su máxima aproximación y el instante en que Sabine oiga el ruido del avión de Marcel?

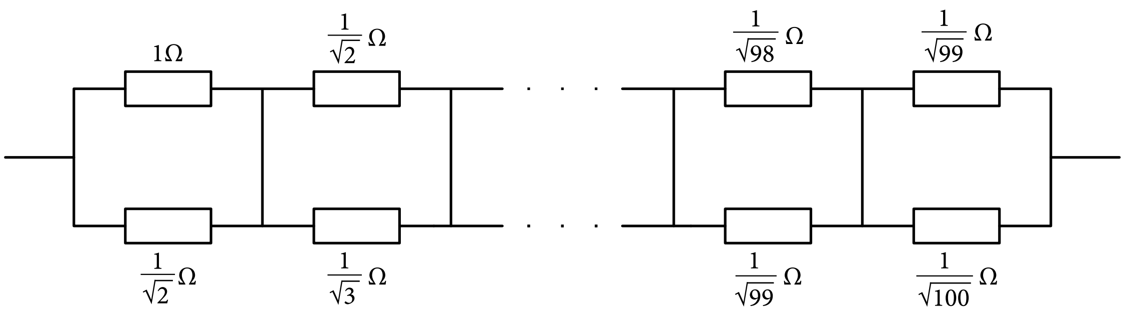
Velocidad del sonido es Mach 1 = 1 km *en* 3 s.

22 Marcel y Sabine se van de luna de miel en globo. Sin embargo, su presupuesto ya es bastante ajustado y, aunque Marcel ha conseguido que le presten un globo gratis, necesita llenarlo con algo. El helio es caro, así que tendrán que conformarse con hidrógeno obtenido electrolíticamente.

¿Cuánto les costará llenar el globo, si el globo vacío con los recién casados tiene una masa de 1000 kg, el precio de la electricidad para las parejas jóvenes es de 0.20 €/kWh , y la eficiencia global del proceso es de 50 %? La combustión del hidrógeno con el oxígeno produce 285.8 kJ/mol de energía.

La luna de miel tendrá lugar a temperatura y presión estándar. La presión en el interior del globo es igual a la presión exterior.

23 Sam encontró una enorme caja de varias resistencias y una larga bobina de alambre perfectamente conductor y construyó una larga escalera con ellos. Entre cada par de peldaños, hay una resistencia de valor de $\frac{1}{\sqrt{n}}\,\Omega$ en un lado y una resistencia de $\frac{1}{\sqrt{n+1}}\,\Omega$ en el otro lado, para n entre 1 y 99.



¿Cuál es la resistencia total entre los extremos de la escalera?

24 La altura del nivel del mar influida por la marea en la ciudad francesa de Saint-Malo alrededor de la luna llena puede modelarse como $A \cdot \sin(\omega t)$, donde A es la amplitud y ω es la frecuencia tal que la marea se produce cada 12 horas.

Los marineros del barco Santiano estarían interesados en saber si podrán atracar cuando vuelvan a casa. El problema es que el barco sólo puede fondear en el muelle si la marea es como máximo $A/3$ más baja que en la pleamar… y los marineros no saben a qué hora se produce la pleamar.

¿Cuál es la probabilidad de que puedan fondear en el muelle, es decir, que lleguen a la hora en que pueden fondear?

25 Max y Moritz miden las distancias entre lugares de la Tierra de formas diferentes: Max, piloto, da la distancia como la longitud de la curva más corta a lo largo de la superficie del planeta; mientras que Moritz, atracador de bancos, da la distancia como una línea recta, aunque pase por debajo de la superficie.

Cuando midieron la distancia entre el aeropuerto y el banco, descubrieron que la diferencia entre las longitudes que habían medido era exactamente 1 m. ¿Qué distancia midió Max?

La Tierra es una bola con radio de 6371 km.

26 Un «elefante» se balancea en un columpio suspendido por cadenas con una longitud de L. En la posición extrema, cuando se detiene por un instante, siente una aceleración de $0.5\,g$.

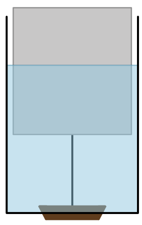
¿Qué aceleración sentirá en la posición más baja?

27 Kate tiene un nuevo collar de metal: tiene forma de bucle circular con radio r y resistencia lineal λ. Durante sus vacaciones, se subió a un tiovivo en el parque infantil de su barrio. Se sentó a una distancia R del eje de rotación y comenzó a girar alrededor de él a velocidad angular ω. En ese momento, se dio cuenta de que unos granujas habían colocado un enorme imán debajo del tiovivo, creando un campo magnético de inducción uniforme B y dirigido en la dirección del eje de rotación.

Kate empezó a preocuparse por su collar: ¿qué corriente se induce en él? Supongamos que Kate está congelada de miedo y permanece en un mismo sitio. Está sentada en una posición tal que el collar forma un ángulo de 30° con el plano horizontal.

28 Lucy ha ideado un ingenioso dispositivo para dosificar correctamente el agua a sus plantas. El dosificador consiste en un recipiente cilíndrico suficientemente alto con un área de base S, que tiene una abertura circular en el centro de su fondo que se extiende hasta la mitad de su radio, cubierta por un tapón sin masa. Dentro del recipiente hay un cilindro más pequeño con una densidad de 500 kg/m^3, una superficie de base de $0.99S$ y una altura de H, que está unido al tapón por la parte inferior mediante un cordón. Después de llenar el recipiente con una cierta cantidad crítica de agua, el cilindro interior levanta el tapón del fondo, impidiendo que se siga añadiendo agua al recipiente.

¿Cuál es el menor y el mayor volumen de agua que Lucy puede dosificar con este dispositivo de medición, si puede ajustar la longitud de la cuerda a cualquier longitud deseada? El volumen del cilindro interior es 1 l.



29 El despacho de Dan es un desastre. Lo único que puede llamarte la atención al caminar es su preciado juguete: un par de péndulos.

Cuando se desplazan simultáneamente de su posición de equilibrio, su movimiento tiene las siguientes características:

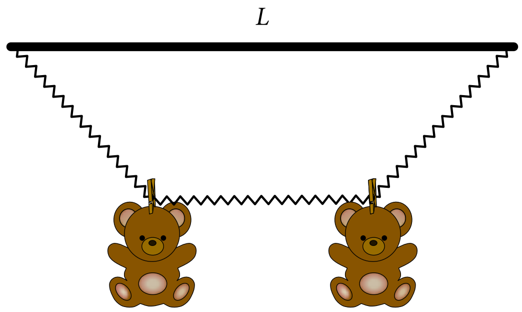
- exactamente con cada tercer paso del péndulo más lento por la posición extrema de la derecha, se encuentra allí con el más rápido, y

- exactamente con cada quinto paso del péndulo más rápido por la posición extrema de la izquierda, se encuentra allí con el más lento.

La longitud de la cuerda del péndulo más largo es 10 cm. ¿Cuál es la longitud de la cuerda del péndulo más corto?

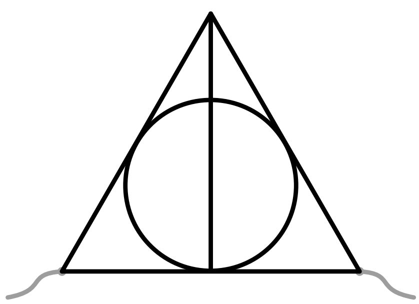
30 Penny necesita secar dos peluches recién lavados, cada uno con una masa de m. Por desgracia, durante su primer intento de colgarlos en una cuerda del tendedero, éste se rompió. Pero Penny fue demasiado lista para su propio bien: estiró un muelle con longitud de reposo cero y rigidez k sobre el marco horizontal ahora vacío del tendedero con anchura L. Luego colgó los peluches a un tercio y dos tercios de la longitud del muelle, que se hundió bajo su peso.

¿Cuánto más bajo estará el centro del muelle con respecto a sus extremos?



31 Sarah vuelve a sacar su colgante favorito de las Reliquias de la Muerte. Esta vez se interesa por su resistencia eléctrica. El colgante de Sarah está formado por un triángulo equilátero de lado a, un círculo inscrito en su interior y una de sus alturas. La resistencia lineal de todos sus componentes es λ.

¿Cuál es la resistencia entre los dos vértices del triángulo que no están tocados por la varita (altura del triángulo)?



32 El Señor Oscuro™ ha pasado algún tiempo de calidad en el vacío, y ahora regresa al mundo de los mortales con un nuevo hechizo de magia negra: apagar la gravedad. De momento sólo puede suspenderla durante un breve periodo de tiempo, pero seguramente debe estar planeando algo mucho mayor.

Ahora te pregunta (por puro interés académico, claro): ¿durante cuánto tiempo necesitaría apagar la gravedad del Sol para que la Tierra volara hasta el infinito?

33 Ellie cogió una vieja plancha doble para sus experimentos culinarios. A pesar de sus expectativas, la placa funcionó muy bien y, tras enchufarla, se calentó a una temperatura astronómica de 250 °C. Sin embargo, a esas temperaturas, la comida se carbonizaba con facilidad. Por desgracia, los mandos de la vieja placa ya no funcionaban, así que Ellie tuvo que improvisar. Cogió un pulverizador con un líquido refrigerante especial a una temperatura de 20 °C y lo aplicó a la placa caliente con un caudal volumétrico constante. ¿Cuál debe ser el caudal volumétrico para que la temperatura descienda a un valor aceptable de 150 °C? La superficie de la placa de cocción es de $0.1\ \mathrm{m}^2$, y tras años de uso intensivo está absoluta e irremediablemente negra.

El líquido especial tiene una capacidad calorífica específica y una densidad constantes en una amplia gama de temperaturas, que son iguales a la capacidad calorífica específica y la densidad del agua a temperatura ambiente. También hierve a una temperatura de 100 °C*, igual que el agua, y tiene el mismo calor latente específico de vaporización.*

34 Jacob lleva una preciosa carga en su coche: una tarta de boda en forma de paraboloide de revolución con altura H y una base de radio R. Al acercarse a un cruce, el semáforo se pone en rojo de repente, lo que le obliga a frenar en seco. ¿Cuál es la mayor deceleración al frenar que puede aplicar para que el pastel no vuelque?

Ignora los efectos distintos de la deceleración.

35 Tras una larga y brillante vida, nuestro Sol morirá. El proceso de muerte es el siguiente: desprenderá una capa de material de su superficie hacia el espacio bajo la influencia de fuerzas radiales.

Consideremos que el Sol es una esfera con un periodo de rotación de 28 días, formada por un núcleo homogéneo y una envoltura exterior homogénea. La relación de masas entre el núcleo y la envoltura es 1 : 1 y la relación de sus densidades es 63 : 1. Cuando el Sol muera, se desprenderá por completo de su envoltura y el núcleo colapsará en una enana blanca, una esfera homogénea con un radio de 5000 km.

¿Cuál será el período de rotación de la enana blanca?

36 El flujo de neutrinos solares a través de la Tierra es de aproximadamente $10^{15}\ \mathrm{m}^{-2}\cdot\mathrm{s}^{-1}$. Dirk construyó un enorme detector en forma de cubo con un volumen de $1000\ \mathrm{m}^3$, lo llenó de agua y descubrió que, por término medio, se captura en él un neutrino cada segundo.

¿Cuál es la trayectoria libre media de un neutrino en el agua?

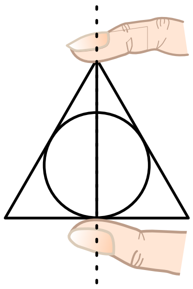
37 Tras haber visto las olimpiadas el tiro con pistola nos ha fascinado y hemos decidido hacer una de aire comprimido. El mecanismo de dicha arma consiste en un cilindro con un volument $V = 100\ \mathrm{cm}^3$ lleno de aire a presión atmosférica y cerrado por un pistón de masa $M = 1$ g. Antes del disparo el mecanismo de disparo se comprime hasta un volumen $\frac{V}{16}$. Después se coloca un proyectil de masa $m = 2$ g enfrente del pistón. Cuando se aprieta el gatillo el pistón se libera y el aire se expande, empujando el proyectil.

Para probar la pistola hemos contratado a dos tiradores: Yusuf Y Kim. El primero llegó, cargó el arma, disparó y se marchó. Cuando la segunda llegó tuvimos mucho tiempo de preparación. Entre la carga y el disparo se tapó el ojo izquierdo, para ver solo con el derecho el objetivo. En este se colocó otra cubierta con un pequeño agujero para ver muy nítidamente. A lo largo de esta preparación el aire del pistón se ha enfriado. Ahora nos interesa la proporción entre las velocidades de los proyectiles disparados por Yusuf y Kim.

El trabajo realizado por el gas en un proceso adiabático es $W = \frac{p_0 V_0 - p_1 V_1}{\kappa - 1}$*, donde el índice* 0 *hace referencia al estado inicial y el* 1*, al final. El aire es un gas ideal diatómico con* $\kappa = 1.4$.

38 Sarah cogió el colgante de las Reliquias de la Muerte por tercera vez. Lo sujetó entre el pulgar y el índice, de forma que la Varita de Saúco quedara directamente sobre la línea que unía estos dos dedos, y luego lo hizo girar. ¿Cuál era el momento de inercia del colgante con respecto a ese eje?

Geométricamente, el colgante de Sarah está formado por un triángulo equilátero de lado a, una circunferencia inscrita y una de sus alturas (Varita de Sauco). La densidad lineal de todos sus componentes es λ.

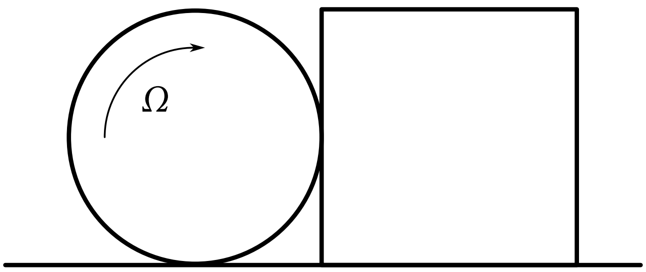


39 El día de su cumpleaños, la abuela Julie tiene que oír el Cumpleaños Feliz de toda su familia. Cada año tiene que oírlo a un nivel de intensidad sonora (medido en decibelios) igual a su edad en años, y tuvimos que esforzarnos cada vez más. El año pasado no pudimos cantar más alto, así que recurrimos a la ayuda de nuestro ruidoso vecino, que es capaz de cantar con un nivel de intensidad sonora de 100 dB a una distancia de 1 m.

Este año descubrimos que ni siquiera la ayuda de nuestro vecino sería suficiente. En su lugar, hemos movido el sillón de la abuela Julie 30 cm más cerca de los parientes cantores. ¿Qué edad tiene la abuela Julie este año?

40 Danny el cubo y Johny el cilindro vuelven a pelearse. Sin puños ni piernas, la pelea procede como sigue: Johny gira hasta alcanzar una velocidad angular $\Omega = 23\ \mathrm{s^{-1}}$, se tumba justo al lado de Danny y comienza a empujar en su lado. Danny cúbico es bastante indiferente a ser empujado, pero le interesa saber qué velocidad máxima alcanzará mientras Johny le empuja.

El coeficiente de rozamiento entre Danny y el suelo y entre Danny y Johny es $f = 0.2$, y el coeficiente de rozamiento entre Johny y el suelo es el doble. Todos los bordes de Danny, la altura y el diámetro de Johny son 1 m y ambos pesan 100 kg.



Soluciones

1 We hope you have passed the hidden IQ test and denoted each fruit with some letter. For example B – banana, L – lemon, G – grape, M – watermelon and C – cherry. The gargantuan equation is then simplified to

$$\frac{M(2B)^2}{2C^2 - \frac{1}{2}\frac{B}{G}} = ?\ \text{Wh}. \tag{1.1}$$

We plug the equation $\frac{B}{C^2} = G$ in the denominator. Next, we multiply the whole expression by $\frac{BL}{BL}$, which leads to

$$\frac{M(2B)^2}{2C^2 - \frac{1}{2}\frac{B}{G}} = \frac{4MB^2}{\frac{3B}{2G}} = \frac{8}{3}MBG = \frac{8}{3}\frac{L}{B}\frac{MB^2G}{L}. \tag{1.2}$$

We notice that the last fraction is equal to unity, as can be deduced from one of the equations from the problem statement, and using another equation, $\frac{L}{B} = 2.7$ kJ. That means that fraction of interest is equal to 7.2 kJ. In order to convert it to Wh, we recall that 1 Wh = 3600 J, so the result is 2 Wh.

We firmly believe that from now on you will not complain about letters in equations.

2 Both racers drive at constant speed and therefore at any time they will have covered the same distance. If Pato drives n times around the roundabout, he will have travelled a distance of $n \cdot 100$ m. We require Josef to drive one circle less, so at the same time he has driven the distance of $(n-1) \cdot 120$ m. Since the covered distances must be equal, we know that

$$n \cdot 100\ \text{m} = (n-1) \cdot 120\ \text{m}, \tag{2.1}$$

which holds for $n = 6$.

3 What happens with Martin's coffee? It will start giving away heat to the aluminium container which begins to heat up. The heat transfer stops once both coffee and container have the same temperature, and the heat given away by coffee equals the heat received by the container.

The volume of the inner container is

$$V = \pi r^2 h \doteq 905\ \text{ml}, \tag{3.1}$$

where r is the base radius and h is the container's height. The coffee with density of water therefore weighs $m_k = V\rho_{H_2O} \doteq 905$ g, its specific heat capacity is c_{H_2O} and its temperature went down from T_k to some lower temperature T.

Then there is the container – a cylinder with mass

$$m_{Al} = (2\pi r^2 + 2\pi rh)\,\Delta h \rho_{Al} \doteq 75\ \text{g}, \tag{3.2}$$

where Δh is the wall thickness and ρ_{Al} is the aluminium density. The container with specific heat capacity c_{Al} had its temperature increased from initial T_{Al}, again to temperature T. The calorimetric equation therefore reads

$$m_k c_{H_2O}(T_k - T) = m_{Al} c_{Al}(T - T_{Al}), \tag{3.3}$$

whence

$$T = \frac{m_k c_{H_2O} T_k + m_{Al} c_{Al} T_{Al}}{m_k c_{H_2O} + m_{Al} c_{Al}}. \tag{3.4}$$

The coffee will therefore cool by $T_k - T \doteq 1.3\,°C$.

4 If we want to convert units, for example from metres to kilometres, we usually multiply the value by number one in a suitable form,

$$1 = \frac{1 \text{ new unit}}{x \text{ old units}}. \tag{4.1}$$

For example we can convert 563 m to kilometres by writing

$$563\text{ m} = 563\text{ m} \cdot \underbrace{\frac{1\text{ km}}{1000\text{ m}}}_{1} = \frac{563}{1000}\text{ km} \cdot \frac{\cancel{m}}{\cancel{m}} = 0.563\text{ km}. \tag{4.2}$$

Equivalently we convert $\mathrm{m/s^2}$ to $\mathrm{km/h^2}$. Since 1 h = 3600 s, suitable ones are $\frac{1\text{ km}}{1000\text{ m}}$ and $\frac{3600\text{ s}}{1\text{ h}}$.

The entire computation is as follows:

$$2\text{ m/s}^2 = 2\text{ m/s}^2 \cdot \frac{1\text{ km}}{1000\text{ m}} \cdot \left(\frac{3600\text{ s}}{1\text{ h}}\right)^2 = \frac{2 \cdot 3600^2}{1000} \cdot \frac{\cancel{m}}{\cancel{m}} \cdot \frac{\cancel{s^2}}{\cancel{s^2}} \cdot \text{km/h}^2 = 25\,920\text{ km/h}^2. \tag{4.3}$$

5 As water flows into the unusual glass creation, it follows certain rules:

1. If it can flow somewhere lower, it will.
2. Therefore the free surface must be at the same height everywhere.
3. At the beginning, there is air everywhere. So if water is to flow somewhere, there must be an opening for the air to escape. If there is no opening, the air will prevent the next water from flowing in.

Gradually, as we pour in water and adhere to these rules, we will reach a filling level as shown in the image 5.1.

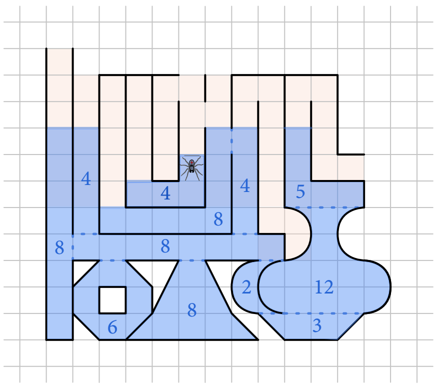


Figura 5.1: *Distribution of water in the container*

All that remains is to calculate the volume of the poured water. If we realize that the cut parts have a volume of 1 l or 0.5 l and the arc-shaped parts complement each other to form whole squares, the volume of the poured water can simply be determined as 72 l.

6 Let us find a solution on a flat Earth first. We draw two circles around Rio, with radii of 17 700 km and 18 600 km, and then try to find a pair of point whose relative distance is largest. These are, quite trivially, any two points on the opposite sides on from Rio and their relative distance is the sum of their radii, or 36 300 km.

On a real spherical Earth it looks similar, however no two points can be further apart than half of its circumference, or 20 000 km – otherwise it would be shorter to turn around and take the other route. The shorter distance is then the complement to the circumference, which is only 40 000 km − 36 300 km = 3700 km.

Alternatively, we may realize that all points at a distance of x from Rio are at a distance of $\pi R - x$ from the antipodal point to Rio (exactly on the other side of the planet). This means they lie on a circle with this radius, and we need to find a pair of most distant points on two circles with radii 1400 km and 2300 km, which is again 3700 km.

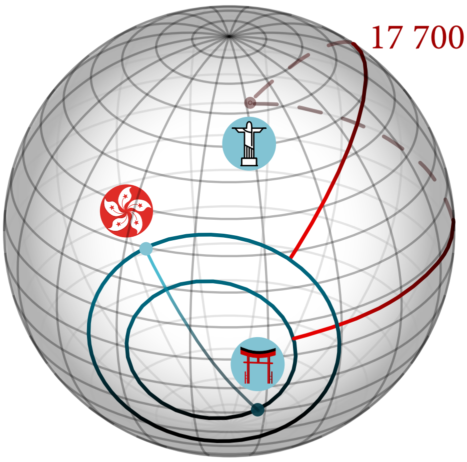


Figura 6.1: *Two possible places where Tokyo (torii) and Hong Kong (flower) might be located. Rio (the statue of Christ) lies on the other side of the Earth.*

7 Let us denote the length of the $L = 300$ m and Thomas's and Matthew's acceleration $a_\mathrm{T} = 8$ m/s and $a_\mathrm{M} = 9$ m/s. Time t_T which it took Thomas to reach the finish line, can be calculated from the equation for motion with constant acceleration

$$L = \frac{1}{2} a_\mathrm{T} t_T^2, \tag{7.1}$$

from where we can express

$$t_\mathrm{T} = \sqrt{\frac{2L}{a_\mathrm{T}}}. \tag{7.2}$$

Similarly, we calculate the time t_M which Matthew took to cross the finish line as

$$t_M = \sqrt{\frac{2L}{a_M}} \tag{7.3}$$

We add $\Delta t = 1$ s to this time and we compare the total times both racers required to reach the finish. After plugging in the values from problem statement we get

$$(t_M + \Delta t) - t_T = 0.505 \text{ s}, \tag{7.4}$$

so Thomas wins by 0.505 s.

8 The numbers are powers of two, so we must decide the truthfulness of all statements.

Hose (1)

The equation of continuity applies: what flows into something must flow out as well. The volumetric flow is constant, and thus a smaller cross-section means a greater velocity. The statement is **false**.

Force acting on a surface of a closed container (2)

Pascal's law could suggest something similar to us. But in addition to the pressure caused by our force, there is also the weight of the liquid, and the pressure it causes is called hydrostatic pressure – this, however, varies with depth, so the statement is **false**.

Body on the surface (4)

The statement is **false**, whether we recall a needle floating on the water surface due to surface tension or a floating iron ship.

Mercury in a measuring cylinder (8)

The pressure at the bottom of the cylinder is hydrostatic pressure, which depends on how deep below the surface this bottom is. By tilting the cylinder, its horizontal cross-section increases, and therefore the liquid level decreases. The hydrostatic pressure thus decreases as well, and the statement is **true**.

Floating ice cube (16)

The submerged part of the ice cube occupies the same volume as water with the same weight – therefore when the cube melts, the newly formed water only occupies the space that the ice cube occupied below the surface. The claim is **false**.[1]

Evaporation (32)

The statement is **false** – even at a lower temperature, the molecules with the highest energy can escape from the surface of the liquid. When boiling point is reached, the liquid evaporates in its entire volume.

[1] If we consider that the temperature of the water decreased due to the melting ice and consequently its volume due to thermal expansion, we would conclude that the level would even drop.

Two bodies (64)

Even though polystyrene foam has a much lower density than wood, a large log and one polystyrene ball can float next to each other – and this is in direct contradiction to the statement in the problem, so it is **false**.

Summing the numbers of the true statements we get the answer 8.

9 When a light ray passes through an optical interface, it refracts according Snell's law. The considered ray passes from a material with refractive index 1 (air) to a material with refractive index n (water) at the angle of incidence 45°. Therefore

$$\frac{\sin 45^\circ}{\sin \alpha} = n. \tag{9.1}$$

If we consider a right triangle determined by the ray, normal to the interface at the place of incidence, and bottom of the pool, according to the Pythagorean theorem, the path length of the ray in the water is $\sqrt{d^2 + h^2}$, where h is depth of the pool and d is distance of the place at the pool bottom which is hit by the ray from the normal. Thus we can write

$$\sin \alpha = \frac{d}{\sqrt{d^2 + h^2}}, \tag{9.2}$$

After substituting into the previous equation, we can express

$$d = \frac{h}{\sqrt{2n^2 - 1}}. \tag{9.3}$$

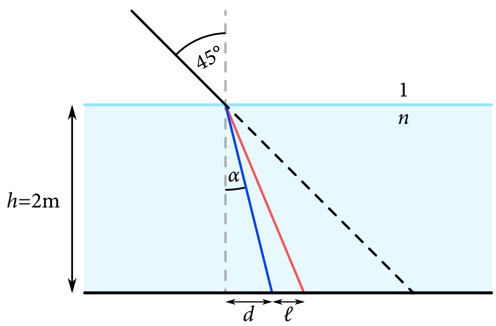


Figura 9.1: *The outermost rays hitting the pool bottom*

Individual colour components of light differ (among other things) by their refractive index. Therefore, each wavelength refracts slightly differently after hitting the interface. From the problem statement we know that the refractive index of water for visible wavelengths lies within the given range – let's denote the limit values n_{min} and n_{max}. Each of them leads to a different value of d. A difference of the ds is the sought length of the rainbow stripe at the bottom of the pool

$$\ell = \left| \frac{h}{\sqrt{2n_{\mathrm{min}}^2 - 1}} - \frac{h}{\sqrt{2n_{\mathrm{max}}^2 - 1}} \right|. \tag{9.4}$$

For the provided numerical values $\ell \approx 13$ mm.

10 In the tank, there is $m_i = 1$ kg of ice with density of $\rho_i = 916\ \mathrm{kg/m^3}$, therefore its volume is $V_i = m_i/\rho_i \doteq 1.092$ l.

If we denote the total volume of the tank as V, the antifreeze mixture occupies a volume of $V_a = V - V_i \doteq 0.908$ l, and therefore its mass is $m_a = (V - V_i)\rho_a \doteq 0.727$ kg. The new freezing temperature is a weighted average according to the mass of the constituents, which is

$$T = \frac{m_i T_i + m_a T_a}{m_i + m_a}, \tag{10.1}$$

where T_i and T_a are the freezing temperatures of water and the antifreeze mixture.

For the values from the problem statement it equals $T \doteq -8.4$ °C.

11 Whenever Adam or the fish bite off a piece of the apple, due to buoyancy it will find a new position, such that the submerged part of the apple will always constitute two thirds of its volume.

Therefore at any given time, both of them have something to bite, however small the remaining piece of the apple is. And because both of them are eating all the time and finish simultaneously, the ratio of eaten parts will be determined by the ratio of their biting rates. Therefore, the answer is $\frac{3}{4}$.

12 Suppose Jeremy's speed is v_0. While braking on flat ground, he decelerates with constant deceleration a, so his speed $v(t)$ changes in time as

$$v(t) = v_0 - at. \tag{12.1}$$

We see that $v(t) = 0$ when $t = v_0/a$, which is the time required to come to a halt. The travelled distance is then

$$s = v_0 t - \frac{1}{2}at^2, \tag{12.2}$$

which means that after time $t = v_0/a$, so before fully stopping, Jeremy has covered the distance

$$s = \frac{1}{2}at^2, \tag{12.3}$$

from where we easily express Jeremy's deceleration

$$a = \frac{2s}{t^2}. \tag{12.4}$$

Using simple trigonometry, we know that if Jeremy is travelling down an inclined plane at an angle α with respect to the horizontal plane, his acceleration is $g \sin\alpha$. If this acceleration is the same as Jeremy's deceleration, he will never stop, so the angle α is

$$\frac{2s}{t^2} = g\sin\alpha \quad \Rightarrow \quad \alpha = \arcsin\frac{2s}{gt^2}. \tag{12.5}$$

13 Even if the circuit may appear strange, Ohm's law is still a good tool to use. In this specific case, the resistance in the circuit depends on the current showed by the ammeter. Specifically, if the ammeter shows there is a current of k amperes, the resistance will be $4k$ Ω.

From the Ohm's law $U = RI$ we obtain

$$9\ \text{V} = 4k\ \Omega \cdot k\ \text{A}, \tag{13.1}$$

and from that,

$$k = \frac{3}{2}. \tag{13.2}$$

The ammeter will indicate $\frac{3}{2}$ A.

14 Since we know the angular speed of the LP record[2], we can determine that after 20 minutes it completed $20 \cdot 33⅓ = 666⅔$ revolutions. In this time, the needle moves along the entire groove, so the groove must twist around the centre of the disc 666⅔ times. Since the diameter of the disc is 30 cm and the groove ends 5 cm from the centre, the grooved part of the disc is 15 cm − 5 cm = 10 cm wide, so the distance between two twists of the groove is approximately

$$w \approx \frac{10\ \text{cm}}{666⅔} = \frac{3}{20}\ \text{mm} = 0.15\ \text{mm}. \tag{14.1}$$

Area S covered by the groove is easily calculated as the difference of the surface of whole disc and the ungrooved part around the centre,

$$S \approx \pi\Big((15\ \text{cm})^2 - (5\ \text{cm})^2\Big) = 200\pi\ \text{cm}^2. \tag{14.2}$$

Finally, we untwist the groove into the shape of a very long rectangle. If we know its surface and width, the length ℓ is equal to their quotient,

$$\ell \approx \frac{S}{w} = \frac{200\pi\ \text{cm}^2}{0.15\ \text{mm}} \approx 418.88\ \text{m} \doteq 419\ \text{m}. \tag{14.3}$$

15 The speed of the jumper during her jump can be decomposed into horizontal and vertical components. The horizontal component is constant and not very interesting to us. Thus, we only need to model the entire situation as a vertical throw. Let's denote the initial speed of the jumper as v_0. This speed changes linearly throughout the motion and this rate of change is the gravitational acceleration g.

At the beginning, the jumper had a speed of v_0, and at the moment when she passed the level of the diving board downwards, her speed was $-v_0$. The difference in this two speeds is therefore $2v_0$ and the time the jumper spent above the level of the diving board must be $\frac{2v_0}{g}$. This is to be half of the total flight time T, thus

$$T = \frac{4v_0}{g}. \tag{15.1}$$

Now let's express the initial speed v_0 as a function of the height of the diving board, $h = 10$ m. We start from the moment when the jumper passes the level of the diving board downwards For her uniformly accelerated motion downwards we can express h as

$$h = v_0\frac{T}{2} + \frac{1}{2}g\left(\frac{T}{2}\right)^2. \tag{15.2}$$

[2] We hope that Patrick's disc was indeed an LP record.

Finally we express v_0 from equation 15.1 and substitute it into equation 15.2. This simplifies to

$$h = \frac{gT^2}{4}, \tag{15.3}$$

from which we can easily express

$$T = 2\sqrt{\frac{h}{g}} \approx 2\ \mathrm{s}. \tag{15.4}$$

16 First, let's consider what the geometry of the necklace looks like. The vertical line segment (the Elder Wand) forms the median, the height, and also the angle bisector of the equilateral triangle (the Invisibility Cloak). We can calculate its length from the Pythagorean theorem as $\frac{a\sqrt{3}}{2}$. Since it is an equilateral triangle, all other medians will also be the angle bisectors. Thanks to this, we know that the centroid of the triangle will coincide with the centre of the inscribed circle (the Resurrection Stone). Its radius will also be one-third the length of the median, i.e. $\frac{a}{2\sqrt{3}}$.

From the symmetry of the pendant we know that the centroid will be located on the line segment. We can then calculate the position of the centroid as the average of the distances of the individual centroids weighted by their masses. Thus, we only need to calculate the vertical position of the centroid. For simplicity, we can choose the point from which we measure the distance to the centroid as the origin. Since mass is directly proportional to the length of the wire, the distance of the centroid from the vertex of the triangle will be

$$x = \frac{m_\triangle x_\triangle + m_\circ x_\circ + m_| x_|}{m_\triangle + m_\circ + m_|} = \frac{3a\frac{a}{\sqrt{3}} + \frac{\pi a}{\sqrt{3}}\frac{a}{\sqrt{3}} + \frac{a\sqrt{3}}{2}\frac{a\sqrt{3}}{4}}{3a + \frac{\pi a}{\sqrt{3}} + \frac{a\sqrt{3}}{2}} = \frac{6 + \frac{2\sqrt{3}\pi}{3} + \frac{3\sqrt{3}}{4}}{6\sqrt{3} + 2\pi + 3} a \doteq 0.555a. \tag{16.1}$$

17 The projectile's launch speed is determined by the law of conservation of energy. At the beginning, both bodies are located at a height of L above the ground and are stationary. Their total mechanical energy E_0 is thus composed only of potential energy, which we can choose as we please, for instance with respect to the ground,

$$E_0 = mgL + MgL. \tag{17.1}$$

At the moment when the pole is vertical, both bodies are moving with a speed of v, but the counterweight is on the ground, and the projectile is at a height of $2L$, thus the total mechanical energy is

$$E_1 = mg2L + \frac{1}{2}Mv^2 + \frac{1}{2}mv^2. \tag{17.2}$$

Since mechanical energy is conserved, it holds that $E_0 = E_1$, i.e.,

$$MgL + mgL = mg2L + \frac{1}{2}Mv^2 + \frac{1}{2}mv^2, \tag{17.3}$$

from which we can express the launch speed of the projectile as

$$v = \sqrt{2gL\frac{M-m}{M+m}}. \tag{17.4}$$

This speed has a horizontal direction.

From now on, the projectile moves along a parabola, since in the vertical direction it is accelerated by gravity. We want to know how long it takes to traverse a length of $2L$ to fall to the ground. This will simply be

$$\frac{1}{2}gt^2 = 2L \quad \Rightarrow \quad t = \sqrt{\frac{4L}{g}}. \tag{17.5}$$

In the horizontal direction, the projectile moves at a constant speed v and in time t it travels to the distance of

$$s = vt = 2\sqrt{2\frac{M-m}{M+m}}L. \tag{17.6}$$

18 Let's assume that all the drivers departed from Bratislava at 10:00 – for the sake of simplicity.

We shall denote the lanes A, B, C and D from the slowest to the fastest. Numbers of cars leaving Bratislava in the specific lanes per minute are a, b, c and d. At first, we need to calculate how long the journey in every lane takes: it is respectively 5, 4, 3⅓ and 2.5 hours.

Arthur in lane B arrived in Prague at 14:00. Meanwhile, some cars were arriving in Prague in the slowest (A) lane – these left Bratislava at 09:00. That does mean Arthur overtook all the cars in lane A which had departed from Bratislava between 09:00 and 10:00 – that is $60a$ cars.

How many cars were overtaken by Peter, the fastest driver? Arriving in Prague at 12:30, he had to overtake the cars in lane C, which had left Bratislava between 9:10 and 10:00, that means $50c$ cars. And the problem states this number is 200, therefore $50c = 200$.

If we make similar calculations for both the drivers and all the lanes, we find that the problem can be reduced to this simple series of equations:

$$\begin{aligned} 50c &= 200, \\ 150a + 90b + 50c &= 620, \\ 100a + 40b &= 220. \end{aligned} \tag{18.1}$$

From that, we can isolate $a = 1$, and therefore $60a = 60$ and Arthur overtook 60 cars on his way.

19 Two of the units, *horse length* and *horse ration*, can be transformed to SI directly, as their dimensions are metres and reciprocal seconds respectively:

- *horse length* ζ is 8 ft ≐ 2.438 m and
- *horse ration* δ is $\frac{1.7\ \mathrm{kg}}{100\ \mathrm{kg}\cdot 1\ \mathrm{d}}$, or about $1.97 \cdot 10^{-7}\ \mathrm{s}^{-1}$.

A *horsepower* is, as its name hints, a unit of power, and its dimension must be the same as that of a watt. When we plug in the values into its definition, we find out that one *horsepower* ψ corresponds to 735.5 W[3]. In base units, the dimension of a watt is

$$\mathrm{W} = \mathrm{J/s} = \mathrm{N}\cdot\mathrm{m/s} = \mathrm{kg}\cdot\mathrm{m^2/s^3}. \tag{19.1}$$

[3] Or 750 W if we use $g = 10\ \mathrm{m/s^2}$.

To obtain a unit of mass, we need to divide power by a squared unit of length and a cubed unit of reciprocal time. Then

$$1\ \text{horse weight} = \frac{\psi}{\zeta^2\delta^3} \doteq \frac{735.5\ \text{kg}\cdot\text{m}^2\cdot\text{s}^{-3}}{(2.438\ \text{m})^2\cdot(1.97\cdot10^{-7}\ \text{s}^{-1})^3} \doteq 1.624\cdot10^{22}\ \text{kg}. \tag{19.2}$$

20 Regardless of what Billy's motion looks like, two conditions must always hold:

- the total time of his run must be equal to the time Cathy spends on the escalator;
- he must end at the same position as Cathy, so the total number of stairs he had run up and down must be equal.

Since his speed while running up is only one third as fast as when running down, the ratios of total time spent running up and down must be reciprocal, or $6 : 2 = 45 : 15$. Since Cathy will obviously spend 60 seconds standing on the stairs, Billy will be running up for a total of 45 s and down for 15 s.

The total distance he will have run will thus be

$$45\ \text{s}\cdot2\ \text{stairs/s} + 15\ \text{s}\cdot6\ \text{stairs/s} = 180\ \text{stairs}. \tag{20.1}$$

21 At any moment, sound spreads from the moving aircraft in all directions at the speed of sound c with respect to the atmosphere. Since the airplane is moving at a speed v greater than c, the envelope of the generated sound waves (shock wave) has the shape of a cone in the direction of the airplane. with a vertex angle of 2α, for which it holds that $\sin\alpha = \frac{c}{v} = \frac{1}{3}$. The vertex of this cone moves in the direction of the aircraft (together with the source) at the speed v, and its surface spreads in the perpendicular direction at speed c.

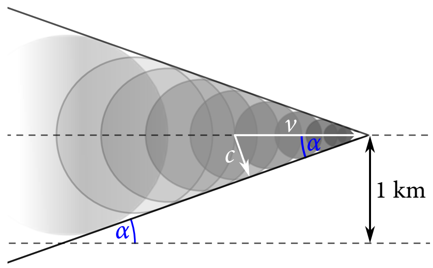


Figura 21.1: *Shock wave from Marc's airplane*

If Sabine with her aircraft were stationary, the sound envelope from Marc's aircraft would reach her only when Marcel was at a distance of

$$1\ \text{km}\cdot\cot\alpha = 1\ \text{km}\cdot\sqrt{\frac{1}{\sin^2\alpha}-1} = 2\sqrt{2}\ \text{km} \tag{21.1}$$

from the point of their closest approach. Since Sabine is moving at a speed v in the opposite direction, both would only manage to reach half the distance of $\sqrt{2}$ km. Marc covers this distance in time

$$\frac{\sqrt{2}\,\mathrm{km}}{1\,\mathrm{km/s}} = \sqrt{2}\,\mathrm{s}. \tag{21.2}$$

22 If the empty balloon with newlyweds weighs M and the hydrogen in it weighs m_{H}, the total gravitational force acting on the inflated balloon will be $(M + m_{\mathrm{H}})g$. For the balloon to float, an upward buoyant force $V\rho_{\mathrm{a}}g$ must act on it, where V is the volume of the balloon and ρ_{a} is the density of air.^[We silently assume the the volume of the newlyweds is negligible compared to this volume. The equation for the equality of forces is

$$(M + m_{\mathrm{H}})g = V\rho_{\mathrm{a}}g, \tag{22.1}$$

from which, substituting $V = \frac{m_{\mathrm{H}}}{\rho_{\mathrm{H}}}$, where ρ_{H} is the density of hydrogen, we get

$$M + m_{\mathrm{H}} = \frac{m_{\mathrm{H}}}{\rho_{\mathrm{H}}}\rho_{\mathrm{a}}, \tag{22.2}$$

and from there we can express the mass of hydrogen as

$$m_{\mathrm{H}} = \frac{M}{\frac{\rho_{\mathrm{a}}}{\rho_{\mathrm{H}}} - 1}. \tag{22.3}$$

Now we can find the density of hydrogen under normal conditions $\rho_{\mathrm{H}} = 0.09\ \mathrm{kg/m^3}$ or utilize the knowledge that under standard conditions, a mole of any gas occupies 22.4 l, meaning it has a molar volume of $V_{\mathrm{m}} = 22.4$ l/mol. Therefore, the density of hydrogen is

$$\rho_{\mathrm{H}} = \frac{m_{\mathrm{H}}}{V} = \frac{m_{\mathrm{H}}}{nV_{\mathrm{m}}} = \frac{M_{\mathrm{H}}}{V_{\mathrm{m}}}, \tag{22.4}$$

where $M_{\mathrm{H}} = 2$ g/mol is the molar mass of molecular hydrogen. Equation 22.3 then transforms into

$$m_{\mathrm{H}} = \frac{M}{\frac{\rho_{\mathrm{a}}V_{\mathrm{m}}}{M_{\mathrm{H}}} - 1} \doteq 73.7\ \mathrm{kg}, \tag{22.5}$$

and therefore the amount of substance of hydrogen is

$$n_{\mathrm{H}} = \frac{M}{\rho_{\mathrm{a}}V_{\mathrm{m}} - M_{\mathrm{H}}} \doteq 36.9\ \mathrm{kmol}. \tag{22.6}$$

This hydrogen was produced electrolytically. The newlyweds are burning it, which is a process opposite to electrolysis. Therefore, if burning a mole of hydrogen releases energy H, it also takes energy H to produce it, at least in an ideal world. However, the newlyweds do not live in such a world, and their electrolysis apparatus has an efficiency η, meaning that to obtain a mole of hydrogen, $\frac{H}{\eta}$ energy is needed, and for n_{H} this is the energy

$$E = \frac{H}{\eta}n_{\mathrm{H}} = \frac{H}{\eta}\frac{M}{\rho_{\mathrm{a}}V_{\mathrm{m}} - M_{\mathrm{H}}}, \tag{22.7}$$

which for the values given is approximately 21 GJ or 5855 kWh. At the newlyweds' electricity price, this costs about 1171 €.

23 We see that between each pair of ranks in the ladder, we have a perfect conductor. Therefore, we can divide the ladder into a series connection of 99 parallel circuits, the total resistance of which we should already be able to calculate. The resistance of the n-th section of the ladder is calculated with the formula for resistance of parallel branches,

$$R_n = \frac{1}{\sqrt{n}}\,\Omega \parallel \frac{1}{\sqrt{n+1}}\,\Omega = \frac{1}{\frac{1}{\frac{1}{\sqrt{n}}\,\Omega} + \frac{1}{\frac{1}{\sqrt{n+1}}\,\Omega}} = \frac{1}{\sqrt{n+1}+\sqrt{n}}\,\Omega. \tag{23.1}$$

Now we know we can reduce the connection to 99 resistors connected in series, and we need to sum their resistances. It does not look simple… fortunately, about the only sensible thing we can do is to expand each value with a suitable one to eliminate the square roots in the denominator:

$$R = \sum_{n=1}^{99} R_n = \sum_{n=1}^{99}\left(\frac{1}{\sqrt{n+1}+\sqrt{n}} \cdot \frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n+1}-\sqrt{n}}\right)\Omega = \sum_{n=1}^{99}\left(\sqrt{n+1}-\sqrt{n}\right)\Omega. \tag{23.2}$$

In this, we identify the so-called *telescopic series*: except for the last positive and the first negative term, everything else cancels each other out, and we get the result

$$\sum_{n=1}^{99}\left(\sqrt{n+1}-\sqrt{n}\right)\Omega = \left(\sqrt{100} - \cancel{\sqrt{99}} + \cancel{\sqrt{99}} - \sqrt{98} + \ldots - \cancel{\sqrt{2}} + \cancel{\sqrt{2}} - \sqrt{1}\right)\Omega = 10\,\Omega - 1\,\Omega = 9\,\Omega. \tag{23.3}$$

24 The sea level will behave over time as a sine, so let's draw one. The sine function has sa period od 2π; however, in our case, it will be scaled to 12 h. The question is, for what portion of the arguments of the function (horizontal axis) is the value greater than $\frac{2}{3}A$. This will not depend on whether the function period is 2π or 12 h – so for simplicity, let's draw one with the period of 2π.

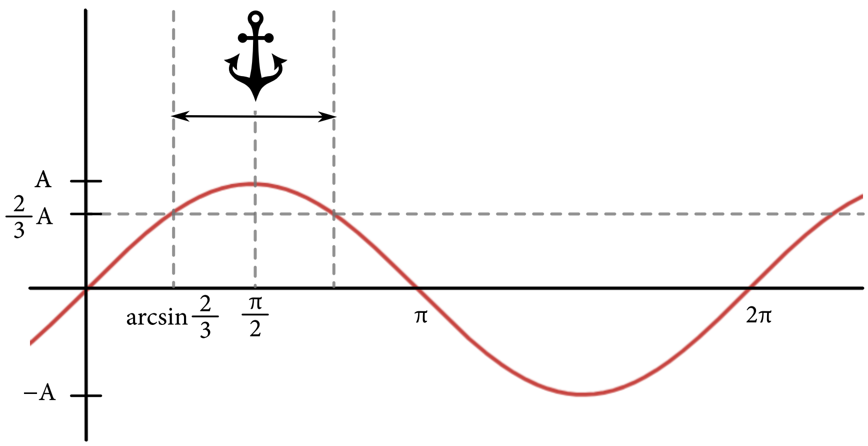


Figura 24.1: *The behavior of the sea level with the time suitable for anchoring marked*

From the image, it is clear that we will be interested in when the function reaches the value $\frac{2}{3}A$. We can calculate it exactly as $\arcsin\left(\frac{2}{3}\right)$.

What portion of one period of length 2π is between $\arcsin\left(\frac{2}{3}\right)$ and $\frac{\pi}{2}$? It is

$$\frac{\frac{\pi}{2} - \arcsin\left(\frac{2}{3}\right)}{2\pi} \tag{24.1}$$

and there are two such pieces, so the total probability is

$$2 \cdot \frac{\frac{\pi}{2} - \arcsin\left(\frac{2}{3}\right)}{2\pi} \doteq 0.267\,72 \doteq 26.8\ \%. \tag{24.2}$$

25 Let's stand in the centre of the Earth and mark the angle between points α. The distance measured by Marcel is $R\alpha$ and the distance measured by Max can be easily calculated as the base of an isosceles triangle, specifically

$$2R \sin\frac{\alpha}{2}. \tag{25.1}$$

These two distances should differ by $\Delta\ell = 1\ \mathrm{m}$. From this, we get the equation

$$R\alpha = 2R \sin\frac{\alpha}{2} + \Delta\ell. \tag{25.2}$$

We would like to solve this equation for α. This is not at all simple! In fact, it is analytically impossible! We will therefore have to settle for an approximate solution, for example through numerical binary search or using Taylor expansion. For the sine function, it holds that

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots. \tag{25.3}$$

Since $\Delta\ell$ is small, we should expect α to be small as well. We will only take the first non-trivial term and get

$$R\alpha = R\alpha - R\frac{\alpha^3}{24} + \Delta\ell, \tag{25.4}$$

from which we manipulate to express

$$R\alpha = R\sqrt[3]{\frac{24\,\Delta\ell}{R}} = \sqrt[3]{24R^2\,\Delta\ell}. \tag{25.5}$$

This is exactly the sought distance of Moritz. Substituting gives 99.2 km.

26 Only two forces are acting on Adam in this problem: gravity, with magnitude mg and tensile force from the chains of the swing, whose magnitude can change, but we know that it is always centripetal – in the direction towards the hinge. The g-force which Adam feels is always equal to the sum of all contact forces, which is in this case means only the tensile strength from the chains. All we need to do is to determine its magnitude at the lowest point.

In the left- and top-most point of his trajectory Adam is not moving at all. Since he does not stay there, he must be subject to some force; and since he is bound to the arc, the direction of this force must be tangential and it must point back towards the lowest point. At the same time, the tensile force must be $m \cdot 0.5\,g$, and the gravity is mg as always.

If we sketch these acting forces for a general angle of maximal deflection α, we see that the magnitude of the centripetal force is $mg\cos\alpha$. This means that in Adam's case α must be 60°, as $\cos 60° = 0.5$.

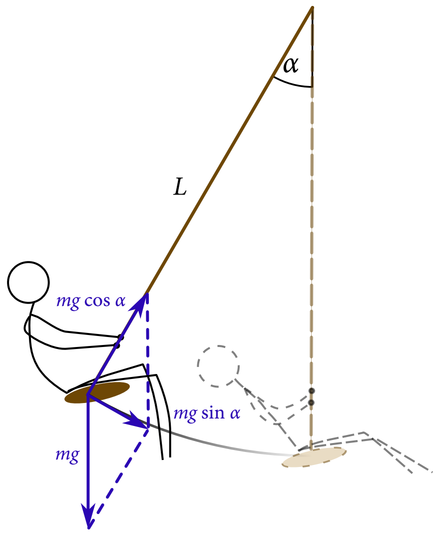


Figura 26.1: *Forces acting on Adam at maximal deflection*

Now we need to calculate Adam's speed at the lowest point. This comes from transforming the difference in potential energies between the highest and lowest points of his trajectory to kinetic energy, which is

$$\Delta U = mg(L - L\cos 60°) = mg\frac{L}{2}, \tag{26.1}$$

and so

$$\frac{mv^2}{2} = mg\frac{L}{2} \quad \Rightarrow \quad v = \sqrt{gL}. \tag{26.2}$$

Finally we find the magnitude of the tensile force from the chains and the corresponding g-force. Its origins are, again, twofold: centripetal force, which ensures that Adam will remain bound to the arc, and the reaction of the swing to Adam's gravity. Their sum is

$$F = ma = \frac{mv^2}{L} + mg = mg + mg = 2mg, \tag{26.3}$$

and the magnitude of apparent acceleration that Adam feels will thus be

$$a = \frac{2mg}{m} = 2\,g. \tag{26.4}$$

27 Faraday's law of electromagnetic induction states that the induced voltage is equal to the negative rate of change of the magnetic flux through a loop. Magnetic flux is calculated as $\Phi = BS\cos\theta$, where B is the magnitude of the magnetic field passing through area S, entering at an angle θ.

In our case, S is the inner area of the necklace. It is evidently on Kate's neck, and thus it has the shape of her neck, which does not change. The magnitude of magnetic induction B also does not change, according to the problem statement. And since Kate rotates only around the axis of rotation of the merry-go-round, the angle θ does not change either.

All of this means that the magnetic flux through Kate's necklace does not change over time, and therefore no voltage is induced in it, and consequently also no current.

28 Let us denote the buoy's base area S_b, its density ρ_b and its volume V_b; the opening's area S_o and the rope's length ℓ. Let the water level reach height h in the limiting situation. Rhe submerged part of the buoy is $v = h - \ell$ tall. The forces acting on the buoy must be balanced, therefore

$$F_G + F = F_{\mathrm{vz}}, \tag{28.1}$$

where F_G is gravity, F_{vz} is buoyancy and F is a force by which the rope pulls on the buoy. In the limiting situation, the force due to the rope is equal to the force due to pressure by which the plug is pushed to the opening,[4], i.e. $F = p_h S_o$, where p_h is hydrostatic pressure near the bottom. After substituting all forces in, we obtain

$$S_b H \rho_b g + h \rho g S_o = S_b (h - \ell) \rho g, \tag{28.2}$$

where ρ is water density.

The least water volume which can be poured in, apparently corresponds to the situation in which the buoy is tied to the bottom as tight as possible, thus $\ell \to 0$. Then we obtain

$$h = \frac{S_b}{S_b - S_o} \frac{\rho_b}{\rho} H \tag{28.3}$$

and the corresponding volume

$$V_{\min} = (S - S_b) h = \frac{S - S_b}{S_b - S_o} \frac{\rho_b}{\rho} V_b. \tag{28.4}$$

The largest possible water volume which can be poured in corresponds to the situation in which the plug is released just in the moment when the buoy is completely submerged, thus when $h = H + \ell$. It happens when the rope's length is

$$\ell = \left(\frac{S_b}{S_o} \frac{\rho - \rho_b}{\rho} - 1 \right) H, \tag{28.5}$$

which corresponds to volume

$$V_{\max} = S\ell + (S - S_b) H = \left[\frac{S}{S_o} \left(1 - \frac{\rho_b}{\rho} \right) - 1 \right] V_b. \tag{28.6}$$

According to the problem statement, $S_o = \frac{S}{4}$ and $S_b = 0.99S$. For the provided numeric values, it yields minimum volume $V_{\min} = \frac{1}{148}\,\mathrm{l}$ and maximum volume $V_{\max} = 1\,\mathrm{l}$.

[4] If it were any larger, it would pull the plug out.

29 The problem statement tells us that during every third passage of the slower pendulum with period T_1 at the extreme right position, it meets the faster one with period T_2 (where $T_1 > T_2$). Between these two moments, the slower pendulum executes three periods and the faster some number of n periods, thus

$$3T_1 = nT_2. \tag{29.1}$$

The second important piece of information is that between these moments, the faster pendulum executes five periods and the slower one some number of m periods, so

$$mT_1 = 5T_2. \tag{29.2}$$

It does not matter at all whether the pendula meet on the left or right. They both perform a simple harmonic motion, and if they meet like this on the left and right, they will meet on both sides after completing five and three periods, respectively. Therefore, it is also unimportant whether Dan initially displaces the pendula to the left or right.

If we divide equations 29.1 and 29.2, we get

$$\frac{3}{m} = \frac{n}{5} \quad \Rightarrow \quad mn = 15. \tag{29.3}$$

One solution to this equation is $m = 15$, $n = 1$, but this solution does not make sense – from the first paragraph we know that if the slower pendulum moves for three periods and the faster n periods, then n must be greater than 3. The second solution to equation 29.3 is $m = 3$, $n = 5$, which meets our requirements. This means that the pendula have periods with lengths in ratio of $3 : 5$.

The period of a pendulum with string length ℓ is

$$T = 2\pi\sqrt{\frac{\ell}{g}}. \tag{29.4}$$

If we write this for our pendula with lengths ℓ_1 and ℓ_2 and know the ratio of the periods, we have

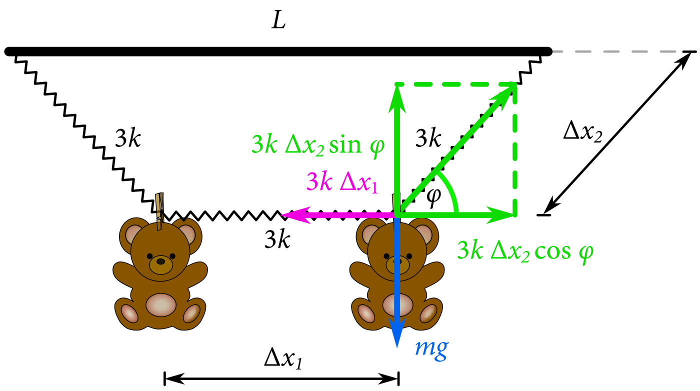
$$\frac{T_1}{T_2} = \sqrt{\frac{\ell_1}{\ell_2}}. \tag{29.5}$$

From the problem statement we know $\ell_1 = 10$ cm, and from equation 29.5 we express

$$\ell_2 = \ell_1\left(\frac{T_2}{T_1}\right)^2, \tag{29.6}$$

which for the ratio $T_2 : T_1 = 3 : 5$ gives us the result $\ell_2 = 3.6$ cm.

30 First, let's realize that if we have a spring with stiffness k and divide it into three equal pieces, each piece will have stiffness $3k$: if we stretch the original spring with a force F, each of its thirds will stretch by one third of the total stretch. However, the same force F acts on each third, which gives us triple stiffness.



At the point where the right plushies is hung, three forces act together. The first, gravity, has a magnitude of mg and points directly downward. The second, from the part of the spring between the plushiesh, points directly to the left and has a magnitude of $3k\,\Delta x_1$. The third generally forms an angle φ with the horizontal direction and has a magnitude of $3k\,\Delta x_2$. We can resolve the third force into a horizontal component $3k\,\Delta x_2 \cos\varphi$ and a vertical component $3k\,\Delta x_2 \sin\varphi$, where we know that the vertical component must equal mg and the horizontal one $3k\,\Delta x_1$. Notice that the distance the plushies drop is exactly $\Delta x_2 \sin\varphi$. We can express this value from the equality of vertical forces as

$$\Delta x_2 \sin\varphi = \frac{mg}{3k}. \qquad (30.1)$$

31 As we have already shown in the sample solution of the task 16, the height of the triangle is $\frac{a\sqrt{3}}{2}$, the radius of the circle is $\frac{a}{2\sqrt{3}}$, and its centre is identical with the centroid of the triangle.

First, we will determine that if we connected a voltage source to the points between which we measure the resistance, no current will flow through the height of the triangle (the base rod). If we connected the voltage source in reverse, the current should flow in the opposite direction. However, we find that the configuration looks exactly the same as before, so the currents must flow the same way. This means that $I = -I$, which is only valid for $I = 0$.

By similar reasoning, we can determine that no current will flow between the circle and the triangle at their bottom point of contact. Since no current will flow through this node, we can disconnect it and simplify the scheme even further. After redrawing, we obtain the scheme shown in figure [@-fig:deathly-hallow-2:sol].

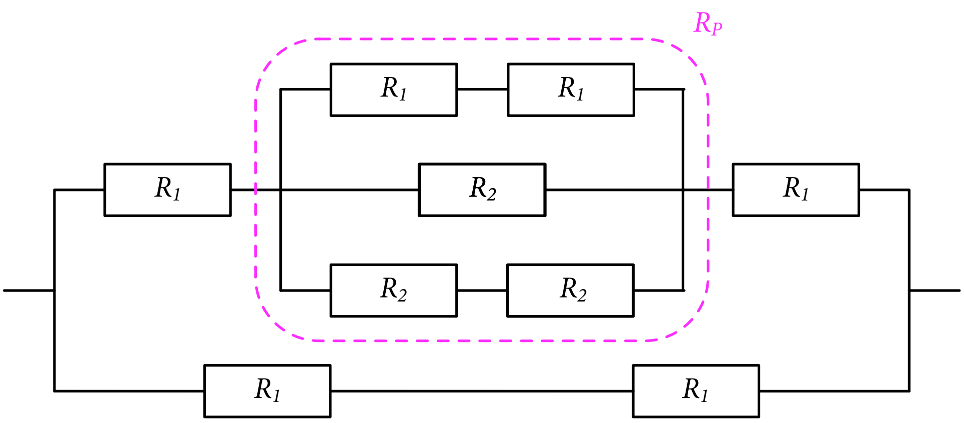


Figura 31.1: *Redrawing of the resistance network in Sarah's pendant*

We used the notation $R_1 = \frac{\lambda a}{2}$ for the resistance of half the side of the triangle and $R_2 = \frac{a\pi\lambda}{3\sqrt{3}}$ for the resistance of one-third of the circle. This is just a series-parallel connection of resistors, whose equivalent resistance we can calculate according to the known rules:

- the resistance of series-connected resistors is the sum of their resistances;
- the reciprocal value of the resistance of parallel-connected resistors is the sum of the reciprocals of their resistances.

If we denote the resistance of the trio of parallel branches as R_P, the following must be true:

$$\frac{1}{R_P} = \frac{1}{2R_1} + \frac{1}{R_2} + \frac{1}{2R_2} = \frac{3R_1 + R_2}{2R_1R_2} \quad \Rightarrow \quad R_P = \frac{2R_1R_2}{3R_1 + R_2}. \tag{31.1}$$

Subsequently, for the total resistance of the pendant we know that

$$\frac{1}{R} = \frac{1}{2R_1} + \frac{1}{2R_1 + \frac{2R_1R_2}{3R_1+R_2}} = \frac{6R_1 + 3R_2}{2R_1(3R_1 + 2R_2)} \quad \Rightarrow \quad R = \frac{2}{3}\frac{3R_1 + 2R_2}{2R_1 + R_2}R_1 = \frac{\lambda a}{6}\frac{9\sqrt{3} + 4\pi}{3\sqrt{3} + \pi} \doteq 0.563\lambda a. \tag{31.2}$$

32 When the gravitational force from the Sun disappears, the Earth will continue moving in a straight line at constant speed, equal to its original orbital speed. The trajectory will be tangent to the original orbit. If the gravity is turned off for long enough, it will certainly move too far to be captured by the Sun again. Between these extremes it will remain in a newer, more eccentric orbit.

The two different fates are separated by a critical situation in which the total mechanical energy after turning the gravity on again is zero. Since without gravity there is no potential energy here, the kinetic energy will remain constant too. Therefore we only need to find the time needed to get to a point where the kinetic and potential energy (after turning the gravity on again) sum to zero:

$$T + U(r) = 0. \tag{32.1}$$

The potential energy in a central gravitational field is

$$U(r) = -\frac{GM_\odot m_♁}{r}, \tag{32.2}$$

while the kinetic energy $T = \frac{m_♁ v^2}{2}$ remains constant. For a known radius of the orbit d we can express v from the identity of gravitational and centripetal force,

$$\frac{v^2}{d} = \frac{GM_\odot}{d^2} \quad \Rightarrow \quad v^2 = \frac{GM_\odot}{d}. \tag{32.3}$$

After plugging equations 32.2, 32.3 into equation 32.1 and dividing by the mass of the Earth $m_♁$ we get

$$\begin{aligned} \frac{GM_\odot}{r} &= \frac{v^2}{2}, \\ \frac{GM_\odot}{r} &= \frac{GM_\odot}{2d} \quad \Rightarrow \quad r = 2d. \end{aligned} \tag{32.4}$$

The Earth thus has to get at least twice as far as it was orbiting before the curse was cast. However as it is not moving radially, but tangentially at first, to find the length of its trajectory L we will use the Pythagorean theorem $L = \sqrt{(2d)^2 - d^2} = \sqrt{3}d$. Now we could express the orbital speed explicitly, however, it is easier to remember that one orbit takes exactly one year. Therefore

$$v = \frac{2\pi d}{1\ \mathrm{a}} \tag{32.5}$$

and the Dark Lord™ wil need to keep the gravity off for

$$t = \frac{L}{v} = \frac{\sqrt{3}d}{2\pi d} \cdot 1\ \mathrm{a} = \frac{\sqrt{3}}{2\pi}\ \mathrm{a} \doteq 101\ \mathrm{d}, \tag{32.6}$$

or a bit more than three months.

33 Absolutely black double hotplate heated to an astronomical temperature T_0 cools down by radiation. At the equilibrium temperature, it radiates with the power supplied from the electrical grid. That is

$$P = S\sigma T_0^4, \tag{33.1}$$

where S is its area and σ is the Stefan-Boltzmann constant.

When cooling it with a liquid, some portion of this power P' is used for heating and evaporation of the liquid, which we can specifically express as

$$P = S\sigma T_1^4 + P', \tag{33.2}$$

where T_1 is the temperature of the cooled double hotplate. The liquid receives power

$$P' = S\sigma(T_0^4 - T_1^4). \tag{33.3}$$

Over time t, the liquid absorbs heat $Q = P't$, which is utilized for heating and evaporating the liquid of mass m, mass heat capacity c, and specific latent heat of vaporization l by temperature ΔT, which we can specifically write as $Q = mc\,\Delta T + ml$. In our case, $\Delta T = 80$ °C, because the liquid is heated from 20 °C to 100 °C, and then it evaporates and leaves the surface. Thus we have

$$P't = mc\,\Delta T + ml. \tag{33.4}$$

We will write m as $V\rho$ and substitute for P' from equation 33.3, to express the volumetric flow rate

$$\frac{V}{t} = \frac{S\sigma(T_0^4 - T_1^4)}{\rho(c\,\Delta T + l)}. \tag{33.5}$$

For the values from the problem, this is 0.094 ml/s.

34 To answer the question, it is necessary to find the centre of mass of the paraboloid. Choose a coordinate system as indicated in the picture. Surface of the paraboloid can be described by function $y = k\sqrt{x}$. As point $[H; R]$ belongs to the paraboloid's surface, $R = k\sqrt{H}$, from where $k = \frac{R}{\sqrt{H}}$, thus $y = \frac{R}{\sqrt{H}}\sqrt{x}$.

The paraboloid's centre of mass lies on the paraboloid's axis at height v. This height can be found if we divide the paraboloid into reasonably chosen pieces, and the paraboloid's centre of mass will be a weighted average of centres of masses of individual pieces. In this particular case, a reasonable choice are very thin horizontal slices. Consequently, the x coordinate of the paraboloid's centre of mass is

$$x_T = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{\sum_i \sigma \pi y_i^2 \cdot x_i}{M} = \frac{\sum_i \sigma \pi \frac{R^2}{H} x_i \cdot x_i}{M}. \tag{34.1}$$

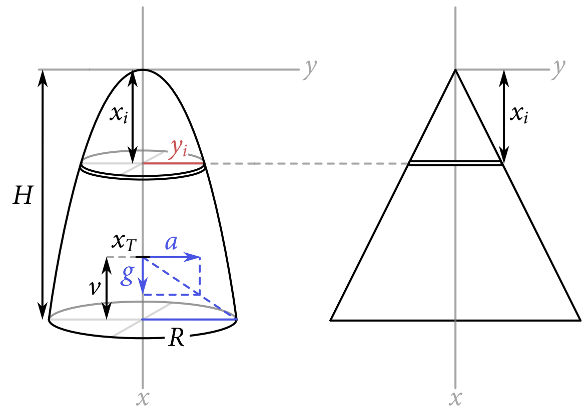


Figura 34.1: *A horizontal slice of a paraboloid and a triangle. In both cases, the mass of the slice increases linearly with the distance from the top, therefore both objects have their centre of mass at the same height.*

Notice that mass of individual pieces $m_i = \sigma \pi \frac{R^2}{H} x_i$ increases linearly with the coordinate. The same property poses also a triangle: if we cut it into slices – the length, and hence the mass of individual slices increases linearly with their distance from the apex.[5] However, we know that triangle's centre of mass lies at one third of its height, therefore the paraboloid must have the same property, hence

$$v = \frac{H}{3}. \tag{34.2}$$

Now, we only need to find a maximum admissible deceleration of the car. For this limit deceleration, the resulting acceleration (car's deceleration plus gravity) directs to the edge of the paraboloid's base. Based on the triangle similarity

$$\frac{a}{g} = \frac{R}{\frac{H}{3}} \quad \Rightarrow \quad a = \frac{3Rg}{H}. \tag{34.3}$$

35 Let's divide the Sun into a core and an outer layer, which is ejected during dying. Because during the ejection process, forces act only in the radial direction, it cannot change the angular momentum of the core. Therefore, we are only interested in the original angular momentum of the core, which will be conserved. We will need its mass and radius for its calculation.

From the density ratio in the problem statement, we know that

$$\rho_e = \frac{\rho_c}{63}, \tag{35.1}$$

[5] This is somewhat analogous to Cavalieri's principle

where ρ_c and ρ_e are the densities of the core and the envelope, respectively.

Furthermore, if we subtract the volume of the core from the volume of the Sun, we obtain the volume of the envelope

$$V_e = \frac{m_e}{\rho_e} = \frac{m_☼}{\rho_☼} - \frac{m_c}{\rho_c}. \tag{35.2}$$

If we use that $m_c = m_e = m_☼/2$ and substitute for ρ_e from equation 35.1, we obtain the density of the core

$$\rho_j = 32\rho_☼ = 32\frac{m_☼}{V_☼} = \frac{24}{\pi}\frac{m_☼}{r_☼^3}. \tag{35.3}$$

We calculate the radius of the spherical core from its mass and density as

$$r_c = \sqrt[3]{\frac{3}{4\pi}\frac{m_c}{\rho_c}} = \frac{r_☼}{4}. \tag{35.4}$$

Let us denote the moment of inertia of the core at the beginning as I_0 (when it has a radius of $r_☼/4$) and at the end as I. The conservation of angular momentum is expressed in the form

$$I_0\omega_0 = I\omega, \tag{35.5}$$

where $\omega_0 = 2\pi/T_0$ is the angular frequency of rotation, and $T_0 = 28$ d is the rotation period at the beginning, where $\omega = 2\pi/T$ is the final angular frequency and T is the required rotation period. By substituting and rearranging, we get

$$T = T_0\frac{I}{I_0}. \tag{35.6}$$

Thus, we only need to calculate the ratio I/I_0. The moment of inertia of a sphere with mass m and radius r is

$$I = \frac{2}{5}mr^2. \tag{35.7}$$

In our case, the core collapses into a smaller sphere, and thus only its radius changes. Therefore, in the ratio I/I_0, everything cancels except for the squares of the radii, so

$$T = T_0\frac{r_{WD}^2}{r_c^2} = 16T_0\frac{r_{WD}^2}{r_☼^2}. \tag{35.8}$$

where $r_{WD} = 5000$ km is the final radius of the white dwarf. After substituting the values, we get $T \doteq 1975$ s.

36 Let us denote the solar neutrino flux $\Phi = 10^{15}/(\mathrm{m^2 \cdot s})$. This is the number of neutrinos that pass through an area of 1 $\mathrm{m^2}$ that is perpendicular to the line towards the Sun in 1 s. We can write

$$\Phi = \frac{\Delta N}{S\,\Delta t}, \tag{36.1}$$

where ΔN is the number of neutrinos, S is the area and Δt is the duration. Furthermore, let us denote the volume of the detector $V = 1000\ \mathrm{m^3}$ and the frequency of interactions between the neutrinos and the water

$R = 1\,\mathrm{s}^{-1}$. This is equal to

$$R = \frac{\Delta N_{\mathrm{int}}}{\Delta t}, \tag{36.2}$$

where ΔN_{int} is the number of neutrinos that react with water within a time interval of length Δt.

Since the neutrinos only interact very rarely, we can assume that the flux Φ is constant throughout the entire volume of the detector. Then we can imagine a huge amount of water through which the neutrinos are flying with a constant flux Φ, but instead of assuming that after a reaction a neutrino disappears (which would decrease the flux), we will assume that it continues its journey. Every neutrino thus flies through the detector undisturbed and interacts, on average, every time is passes distance ℓ.

Now imagine, all at once, the ΔN neutrinos that fly through a certain surface with area S in time Δt – this means, that on average every neutrino interacts exactly once in a volume of $S\ell$. Now we can express the quantity $\frac{R}{V}$, which represents the number of interactions per unit of volume and time, namely

$$\frac{R}{V} = \frac{\Delta N}{S\ell\,\Delta t} = \frac{\Phi}{\ell}. \tag{36.3}$$

From there we can immediately express

$$\ell = \frac{\Phi V}{R}, \tag{36.4}$$

and find the value $\ell = 10^{18}\ \mathrm{m} \doteq 106\ \mathrm{ly}$.

37 Let's start with Yusuf. Since he compressed the piston very quickly, we can assume that the heat exchange between the gas and the environment was negligible. Therefore, it was an adiabatic process. Let us denote the pressure of the air as p_{Y} and the volume of air after compression as $V_0 = \frac{V}{16}$. From the adiabatic law, we have

$$p_{\mathrm{Y}} V_0^{\kappa} = pV^{\kappa}, \tag{37.1}$$

where $p = 101\,325\ \mathrm{Pa}$ is the atmospheric pressure. From this we derive

$$p_{\mathrm{Y}} = p\frac{V^{\kappa}}{V_0^{\kappa}} = 16^{\kappa} p. \tag{37.2}$$

When fired, the gas expands adiabatically back to the original volume V. We can calculate the work done by the gas during the adiabatic process and set this work equal to the kinetic energy of the piston and the projectile,

$$\frac{1}{2}(m+M)v_{\mathrm{Y}}^2 = \frac{p_{\mathrm{Y}} V_0 - pV}{\kappa - 1}, \tag{37.3}$$

where v_{Y} is the speed of Yusuf's projectile. By substituting for p_{Y} from equation 37.2 and V_0, we obtain

$$v_{\mathrm{Y}} = \sqrt{\frac{2}{m+M}\frac{pV}{\kappa-1}\left(16^{\kappa-1}-1\right)} \approx 185.2\ \mathrm{m/s}. \tag{37.4}$$

Now let's look at Kim's shot. She also compressed the piston adiabatically, but then allowed it to cool to the ambient temperature at constant volume. However, since we are not interested in this process, but only its final state, we can treat it as an isothermal compression. Thus, the pressure p_{K} after compression can be calculated as

$$p_K V_0 = pV \quad \Rightarrow \quad p_K = p\frac{V}{V_0} = 16p. \tag{37.5}$$

Here, we must realize that in this case, during the shot, the process is again very quick and thus adiabatic. Atmospheric pressure is reached in the piston before the full volume V is achieved. The projectile is thus accelerated only until it reaches some partial volume V_1, where the pressures are equal (the pressure will thus equal the atmospheric pressure p). The piston will now start to decelerate while the projectile continues on its path. From the adiabatic law, we can derive

$$p_K V_0^\kappa = pV_1^\kappa, \tag{37.6}$$

and by substituting from equation 37.5, we get

$$V_1 = \left(\frac{p_K}{p}\right)^{\frac{1}{\kappa}} V_0 = 16^{\frac{1}{\kappa}-1} V. \tag{37.7}$$

Now we again calculate the work done by the gas as it adiabatically expands from volume V_0 to volume V_1 and set it equal to the kinetic energy of the piston and the projectile,

$$\frac{1}{2}(m+M)v_K^2 = \frac{p_K V_0 - pV_1}{\kappa - 1}. \tag{37.8}$$

By substituting $p_K V_0 = pV$ and V_1 from equation 37.7, we obtain

$$v_K = \sqrt{\frac{2}{m+M}\frac{pV}{\kappa-1}\left(1 - 16^{\frac{1}{\kappa}-1}\right)}, \tag{37.9}$$

which gives 96.1 m/s after substitution.

The quotient of v_Y to v_K is thus

$$\frac{v_Y}{v_K} = \frac{\sqrt{\frac{2}{m+M}\frac{pV}{\kappa-1}(16^{\kappa-1}-1)}}{\sqrt{\frac{2}{m+M}\frac{pV}{\kappa-1}\left(1-16^{\frac{1}{\kappa}-1}\right)}} = \sqrt{\frac{16^{\kappa-1}-1}{1-16^{\frac{1}{\kappa}-1}}},$$

which, after substituting $\kappa = 1.4$, results in $\frac{v_Y}{v_K} \doteq 1.93$.

38 Let's start by solving the geometry. If an equilateral triangle has a side length a, the inscribed circle has a radius $r = \frac{\sqrt{3}}{6}a$, and the height of this triangle is $v = \frac{\sqrt{3}}{2}a$.

The pendant consists of an equilateral triangle with a side length a, a circle with radius r, and a segment of length v. From the problem, we know that the axis of rotation passes through the entire segment, so its moment of inertia is $I_| = 0\ \mathrm{kg\cdot m^2}$.

The moment of inertia of the circle is a greater challenge. We know that the moment of inertia of a body can be calculated by breaking the body into small pieces and summing the contribution of each piece to the total moment of inertia, thus $I = \sum_i m_i\rho_i^2$, where m_i is the mass of the i-th piece and ρ_i is the perpendicular distance of this piece from the axis of rotation. If the axis of rotation passes through the centre of the circle perpendicular to the plane of the circle, its moment of inertia would be

$$I_{\odot} = \sum_i m_i\left(x_i^2 + y_i^2\right) = mr^2. \tag{38.1}$$

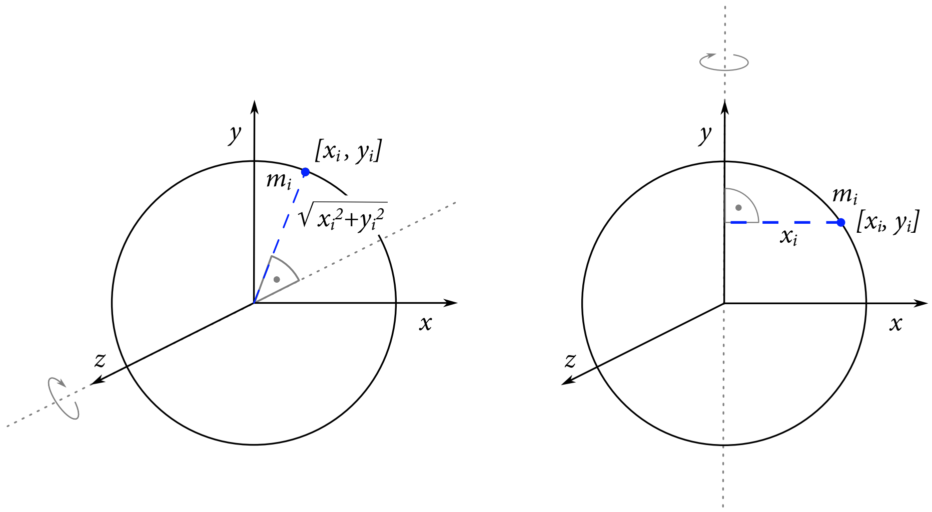


Figura 38.1: *The relationship between the moments of inertia for a circle rotating around two axes passing through its centre: axis perpendicular to its plane and an axis lying in its plane*

However, we have the axis of rotation located in the plane of the circle, thus

$$I_{\varnothing} = \sum_i m_i x_i^2. \tag{38.2}$$

When we realize that we can name the coordinates arbitrarily, obviously $2I_{\varnothing} = I_{\odot}$, which means

$$I_{\varnothing} = \frac{1}{2}mr^2. \tag{38.3}$$

The mass of the circle is $m = \lambda 2\pi r$, therefore

$$I_{\varnothing} = \lambda\pi r^3 = \frac{\sqrt{3}\pi}{72}\lambda a^3. \tag{38.4}$$

We still need the moment of inertia of the triangle. The moment of inertia of a rod perpendicular to the axis of rotation around its centre is $I_{-} = \frac{1}{12}m\ell^2 = \frac{1}{12}\lambda a^3$. When we realize that the moment of inertia of a point mass depends only on the perpendicular distance from the axis of rotation, in the case of a slanted rod we have to calculate the amount of mass of the rod at that distance. Here, this is very simple: mass is distributed homogeneously along their length, and so the moment of inertia of the empty triangle is the same as that of a single rod, but with three times more mass, so $I_{\triangle} = 3I_{-} = \frac{1}{4}\lambda a^3$.

Finally, we simply add the individual contributions to obtain the result

$$I = I_{|} + I_{\varnothing} + I_{\triangle} = \frac{\sqrt{3}\pi + 18}{72}\lambda a^3. \tag{38.5}$$

39 First of all, we must realize what we are going to calculate. Sound intensity level is a quantity which quantifies the energy per unit area and unit time carried by the sound wave. To be more precise, this is quantified by aound intensity I. Sound intensity level L is just a logarithm of ratio of sound intensity to some reference intensity I_0, and it is measured in bels, or in decibels:

$$L = \log\left(\frac{I}{I_0}\right)[\mathrm{B}] = 10\log\left(\frac{I}{I_0}\right)[\mathrm{dB}] \tag{39.1}$$

If a sound source emits acoustic power P, sound intensity at distance r from the source is $I(r) = \frac{P}{4\pi r^2}$, and the corresponding intensity level is

$$L(r) = 10\log\left(\frac{P}{4\pi I_0 r^2}\right) = 10\log(P) - 10\log(4\pi I_0) - 20\log(r). \tag{39.2}$$

Now we can start calculating. According to the problem statement, the intensity level equals Granny Julie's age every year. If Granny Julie celebrates L^{th} birthday this year, her age two years ago was

$$L - 2 = 10\log(P) - 10\log(4\pi I_0) - 20\log(r). \tag{39.3}$$

Last year, our neighbour came to help. If the neighbour can sing with acoustic power Π, granny's age must have been

$$L - 1 = 10\log(P + \Pi) - 10\log(4\pi I_0) - 20\log(r). \tag{39.4}$$

This year, we additionally move Julie's armchair by d towards the singing congratulants, and therefore her age is

$$L = 10\log(P + \Pi) - 10\log(4\pi I_0) - 20\log(r - d). \tag{39.5}$$

Subtracting equation 39.3 from equation 39.4 yields

$$1 = 10\log(P + \Pi) - 10\log(P) \quad \Rightarrow \quad P = \frac{\Pi}{\sqrt[10]{10} - 1} \tag{39.6}$$

and subtracting equation 39.4 from 39.5 yields

$$1 = 20\log(r) - 20\log(r - d) \quad \Rightarrow \quad r = \frac{\sqrt[20]{10}d}{\sqrt[20]{10} - 1}. \tag{39.7}$$

After substituting this into equation 39.3, we obtain

$$L = 2 + 10\log\left(\frac{\Pi}{\sqrt[10]{10} - 1}\right) - 10\log(4\pi I_0) - 20\log\left(\frac{\sqrt[20]{10}d}{\sqrt[20]{10} - 1}\right). \tag{39.8}$$

To determine Granny Julie's age, we must express the acoustic power of the invited neighbour in terms of sound intensity level which he can produce. If he can produce intensity level Λ at distance ρ, we can write

$$\Lambda = 10\log(\Pi) - 10\log(4\pi I_0) - 20\log(\rho). \tag{39.9}$$

After substituting into equation 39.8, we finally obtain

$$L = 2 + \Lambda - 10\log\left(\sqrt[10]{10} - 1\right) + 20\log\left(\frac{\sqrt[20]{10} - 1}{\sqrt[20]{10}} \cdot \frac{\rho}{d}\right), \tag{39.10}$$

which means that, according to the provided numeric values, Granny Julie celebrates her 99$^{\text{th}}$ birthday this year. Congratulations!

40 We will start by drawing all the forces acting on the cube and cylinder.

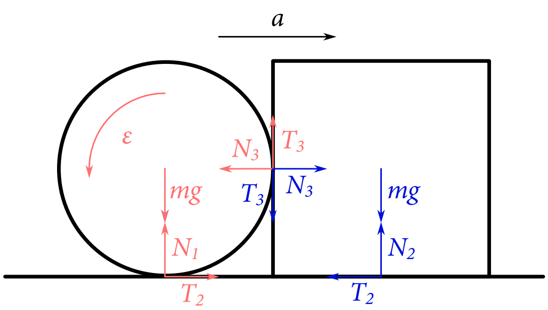


Figura 40.1: *Sketch of forces acting on the cylinder and cube*

Once we have the forces, we can write the motion equations. There will be five in total. Two for the motion of bodies in the vertical direction,

$$\begin{aligned} N_1 + f_3 N_3 &= mg, \\ N_2 - f_3 N_3 &= mg, \end{aligned} \tag{40.1}$$

two for the motion of bodies in the horizontal direction

$$\begin{aligned} ma - f_1 N_1 + N_3 &= 0, \\ ma + f_2 N_2 - N_3 &= 0, \end{aligned} \tag{40.2}$$

and one for the rotation of the cylinder

$$I\varepsilon - f_1 N_1 R - f_3 N_3 R = 0, \tag{40.3}$$

where a is the translational acceleration of the cylinder and cube. These must be equal, otherwise the cylinder would stop pushing the cube, and the fight would be over. $I = \frac{1}{2}mR^2$ is the moment of inertia of the cylinder, and ε is its angular acceleration.

By solving this gargantuan system of equations, we get

$$a = \frac{f(1-4f^2)}{2+f^2}g,$$
$$\varepsilon = \frac{4f+3f^2-4f^3}{2+f^2}\frac{2g}{R}. \tag{40.4}$$

When will the cube have the highest speed? The only force that can accelerate the whole system is the slip of the cylinder. When the cylinder stops slipping, the system will start to slow down due to the friction between the cube and the surface. The cylinder will stop slipping at time τ from the start, for which

$$a\tau - \Omega R + \varepsilon R\tau = 0, \tag{40.5}$$

which is the moment when the circumferential speed of a point on the surface of the cylinder matches its translational speed.

From there, we can express the maximum speed to which Danny is accelerated as

$$v = a\tau = a\frac{\Omega R}{\varepsilon R + a} = \Omega R\frac{1-4f^3}{9+6f-12f^2} \doteq 0.99\ \mathrm{m/s}, \tag{40.6}$$

which is about one meter per second.

Respuestas

1 2 Wh

2 6

3 1.3 °C

4 25 920 km/h^2

5 72 l

6 3700 km

7 Thomas, by 0.505 s

8 8

9 13 mm

10 −8.4 °C

11 $\frac{3}{4}$

12 $\arcsin \frac{2s}{gt^2}$

13 $\frac{3}{2}$ A

14 $\frac{400\pi}{3}$ m $\doteq$ 419 m

15 2 s

16 $\frac{6 + \frac{2\sqrt{3}\pi}{3} + \frac{3\sqrt{3}}{4}}{6\sqrt{3} + 2\pi + 3} a \doteq 0.555a$

17 $L\sqrt{8\dfrac{M-m}{M+m}}$

18 60

19 $1.624 \cdot 10^{22}$ kg, *acepte resultados en el intervalo* $1.62 \cdot 10^{22}$ kg – $1.66 \cdot 10^{22}$ kg.

20 180

21 $\sqrt{2}\,\mathrm{s} \doteq 1.41\,\mathrm{s}$

22 1171 €, *acepte resultados en el intervalo* 1150 € – 1180 €.

23 9 Ω

24 $\dfrac{1}{2} - \dfrac{\arcsin\frac{2}{3}}{\pi} \doteq 26.8\,\%$

25 99.2 km, *acepte resultados en el intervalo* 99 km – 99.3 km.

26 $2\,g$

27 0 A

28 $V_{\mathrm{min}} = \dfrac{1}{148}\,\mathrm{l}$, $V_{\mathrm{max}} = 1\,\mathrm{l}$

29 3.6 cm

30 $\dfrac{mg}{3k}$

31 $\dfrac{a\lambda}{6}\dfrac{4\pi + 9\sqrt{3}}{\pi + 3\sqrt{3}} \doteq 0.563a\lambda$

32 $\dfrac{\sqrt{3}}{2\pi}\,\mathrm{a} \doteq 101\,\mathrm{d} \doteq 8.7 \cdot 10^{6}\,\mathrm{s}$

33 0.094 ml/s

34 $\dfrac{3Rg}{H}$

35 1975 s ≐ 33 min

36 10^{18} m ≐ 106 ly

37 1.93

38 $\dfrac{\sqrt{3}\pi + 18}{72}\lambda a^3$

39 99

40 0.99 m/s

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