Dear readers,

in your hands you are holding the booklet of the 26th volume of Náboj Physics. The booklet contains all the physics problems you could have encountered during the competition this year, as well as the solutions to them, from which you can learn a lot. If you have any difficulty understanding any of them, do not hesitate to contact us; we will gladly clarify everything.

This booklet would not exist without the enormous effort of many people who participated in organising Náboj Physics. Most of us are students of Faculty of Mathematics, Physics and Informatics of Comenius University in Bratislava, and some of us also actively participate in organising the Physics Correspondence Seminar (FKS).

Náboj Physics continues with its international tradition. In the year 2023, Náboj Physics was held in Bratislava, Košice, Prague, Ostrava, Budapest, Gdańsk and Madrid. The results of this international clash can be found on our web site. For the international cooperation, we would like to thank to the local organisers: Patrik Rusnák (Bratislava), Marián Kireš (Košice), Jakub Kliment (Prague), Lenka Plachtová (Ostrava), Ágnes Kis-Tóth (Budapest), Brygida Mielewska and Kamil Żmudziński (Gdańsk) and José Francisco Romero García (Madrid).

In the name of the entire team of organisers, we believe that you enjoyed Náboj Physics in 2023, and we hope that we see each other at Náboj next year. Either as competitors or organisers.

Jaroslav Valovčan
Chief organiser

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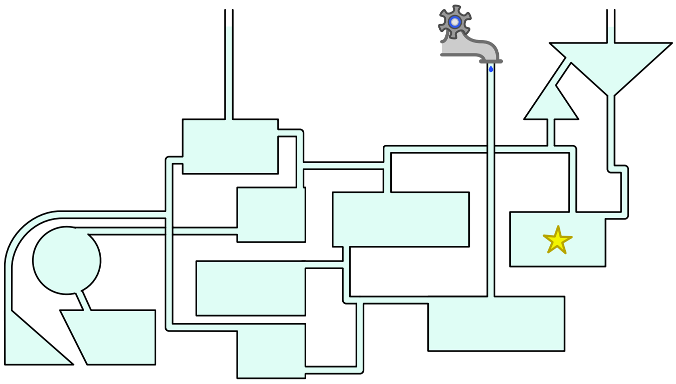
The results, the archive and other information can be found on the web site `https://physics.naboj.org/`.

Problemas

1 Cerca de la casa de Adam se está reconstruyendo una tubería de suministro de agua. Cuando Adam pasó junto a los sudorosos trabajadores, vio una interesante red de tuberías y depósitos de agua, como se muestra a continuación. Los obreros empezaron a llenar de agua la red. ¿Cuántos depósitos estarán llenos de agua en el preciso instante en que el depósito marcado con una estrella esté completamente lleno?



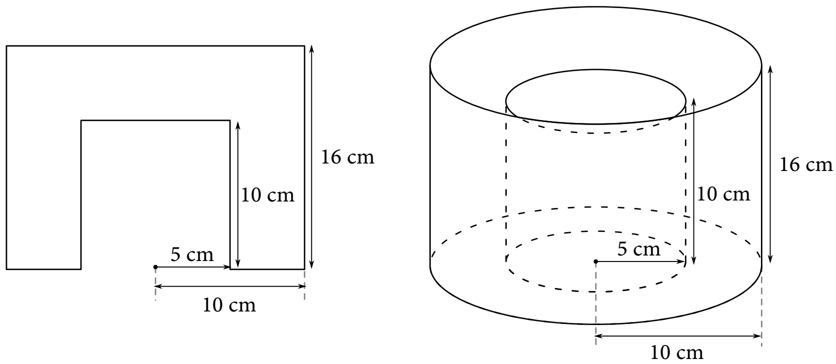
2 Pepito Pérez tiene 100 000 pelos, y cada pelo crece 15 cm al año. ¿Cuánto cambia la longitud total de los pelos de Pepito Pérez en un día?

3 Tomás encendió la televisión para ver una carrera en directo de los Estados Unidos. El comentarista de la carrera remarcó que 200 millas por hora es un campo de fútbol por segundo. Tomás estaba muy impresionado del ingenio del sistema americano de unidades, hasta que lo comprobó y descubrió que el comentarista estaba muy equivocado. ¿Por cuántos campos de fútbol estaba equivocado el comentarista?

Un campo de fútbol americano mide 120 yardas y una milla son 1760 yardas.

4 La inflación ha afectado incluso a la empresa *Ziggo & Marinelli*, líder en la fabricación de galletas molidas. A diferencia de sus competidores, ellos no han aumentado el precio de sus productos; han rediseñado la forma de sus paquetes. El nuevo paquete tiene la forma de un cilindro hueco de radio 10 cm y altura 16 cm. La sección central de su base, un círculo con radio 5 cm, está elevada 10 cm.

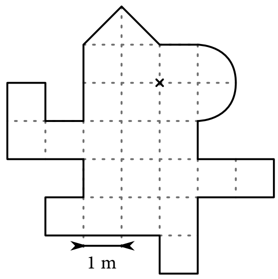
¿Cuál es la densidad de la galleta molida si el paquete entero pesa 350 g y el paquete vacío pesa 40 g?



5 A Jorge le gusta tanto correr… que hasta se compró un smartwatch. Después de un mes de uso intensivo, le felicitó por alcanzar una velocidad de carrera de 17 km/h. Pero ahora está lloviendo afuera y Jorge tiene que correr en una cinta con una longitud de 2 m. Ajusta la velocidad de la cinta a 5 m/s y empieza a correr justo en medio de ella. ¿Cuánto tardará Jorge en caerse de la cinta?

6 Sebastián el pistolero se encuentra en la plaza de un pueblo sin nombre del Lejano Oeste. Sus planes de disfrutar tranquilamente del mediodía se ven frustrados cuando poco a poco un puñado de forajidos lo van acorralando. La fuga del pistolero ahora depende de una distracción. Por suerte, tras la juerga de la noche anterior hay en medio de la plaza un charco enorme de whisky muy inflamable. Sebastián dispara con su revolver al punto marcado en el diagrama con una cruz y el licor se enciende y las llamas comienzan a expandirse a un ritmo de 2 m/s. ¿Cuánto tiempo tardará en estar en llamas todo el charco?

Cada cuadrado de la cuadrícula tiene un lado de 1 m.



7 *Wings for Life* es una carrera benéfica que sigue las siguientes reglas: todos los corredores comienzan a correr al mismo tiempo. Media hora más tarde un coche empieza a seguirlos a 14 km/h y cada media hora acelera instantáneamente a una velocidad mayor. Los corredores adelantados por el coche quedan eliminados.

La mayor distancia jamás alcanzada es de 92,14 km. ¿Cuánto tiempo estuvo corriendo el corredor si las velocidades del coche en los distintos segmentos de media hora son 14, 15, 16, 17, 18, 22, 26, 30 y 34 km/h? Tras alcanzar los 34 km/h el coche deja de acelerar.

8 Dé el resultado de la suma de los números asociados a las afirmaciones que sean ciertas:

1	Los interruptores automáticos (*circuit breakers*) habituales de un apartamento permiten el paso de corrientes nominales de hasta 150 A, más o menos.
2	Las celdas galvánicas son fuentes de corriente continua.
4	Puede existir una fuerza eléctrica entre aislantes.
8	La resistividad de los conductores eléctricos se reduce a medida que se calientan.
16	El culombio es dimensionalmente equivalente a un amperio *entre* un segundo.
32	Las corrientes inferiores a 1 A no son peligrosas para nosotros los humanos.
64	Dos resistencias conectadas en paralelo pueden tener distintas diferencias de potencial.
128	Aunque en las tomas de corriente de la pared suministran corriente alterna, los hervidores eléctricos funcionarían con corriente continua también.

9 Después de su última visita al apartamento de Adam, Eva juró pintar su habitación mientras él estaba fuera. Compró un cubo con 5 litros de pintura con varias inscripciones: aparte de dibujos de la Antártida y alegaciones imprecisas de un blanco inimaginable, se indica el peso de los contenidos, 7,5 kg, y una recomendación de que la pintura debería ser diluida en una razón de 0,5 kg de agua por 1 kg de pintura.

A Eva le da mucha pereza usar una báscula de cocina y prefiere utilizar un cucharón o cazo. ¿En qué proporción **volumétrica** debería diluir la pintura con el agua?

Envíe la razón entre el volumen de pintura y el volumen de agua.

10 Todas las canciones del verano chapuceras recalcan la importancia de mantenerse bien hidratados y frescos en los meses más calurosos. Justino está haciendo una bebida que le preparaba su abuela y que se sirve a 7 °C. Desafortunadamente su grifo solo saca agua a 20 °C. Para enfriarla, añadió bloque de hielo de 1 kg de masa a 0 °C de temperatura.

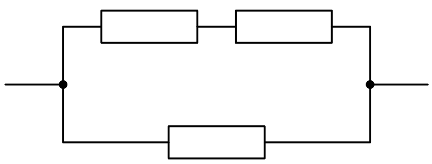
¿Cuál es el volumen de agua a la temperatura deseada que puede producir de esta forma?

11 Por motivos científicos, Matías y Sara han decidido experimentar de primera mano la ingravidez. Planean saltar de un globo aerostático. Cuando alcanzan una altitud de 200 metros, Matías, muy emocionado, saltó. Solo 3 segundos despúes, Sara se dio cuenta de que se le había olvido el paracaídas a su amigo y gritó horrorizada.

¿A qué altitud se encontraba Matías cuando escuchó el grito de Sara?

El globo se encuentra estacionario todo el tiempo. La deceleración debida a la resistencia del aire es despreciable.

12 Cogimos tres resistencias idénticas y medimos su resitencia al conectarlas en serie y en paralelo. La diferencia entre las dos mediciones fue de 8 Ω. ¿Cuál será la resistencia del circuito de la imagen?



13 El bombero Francis está descendiendo en rápel desde un edificio alto. Acaba de llegar al final de una cuerda de 30 m de longitud. Si ahora utiliza sus piernas para alejarse de la pared con todas sus fuerzas, alcanzará una distancia horizontal máxima de 8,4 m de la pared.

¿A qué altura puede saltar Francisco verticalmente en un terreno llano?

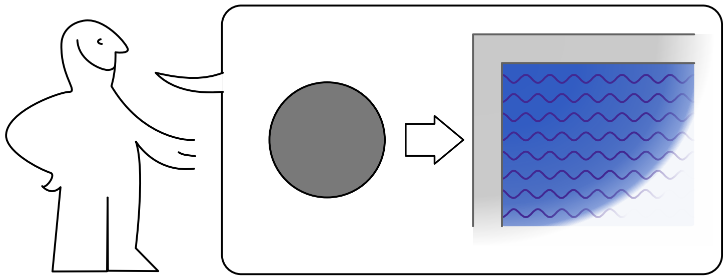
14 Jacobo está yendo demasiado rápido en la autopista. Está notando unas vibraciones que vienen de una de sus ruedas y se está empezando a preocupar un poco. Consideremos, por ejemplo, que una válvula de neumático sencilla tiene una masa de 10 g. Calcule la masa aparente de la válvula que siente el neumático si se encuentra a una distancia de 20 cm del eje de rotación y el coche circula con una rapidez de 500 km/h. Asuma que la distancia de la válvula al eje de rotación es mucho mayor que al borde exterior de la rueda.

La masa aparente de un objeto es la masa que ejercería una fuerza igual cuando se pusiese sobre una báscula en reposo sobre el suelo.

15 Julia está de vacaciones. Su apartamento tiene una piscina rectangular de tamaño $a \times b$. Al llegar, el dueño le informó que la piscina debe cubrirse por las noches. Sin embargo, Julia solo ha encontrado una tapa circular de radio $r \ll a, b$.

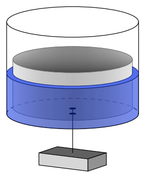
Julia sabe que no la va a poder cubrir totalmente, pero insiste en cubrir la mayor superficie posible.

¿Cúal es la mayor superficie posible de la piscina que puede cubrir sin que la tapa se caiga al agua?



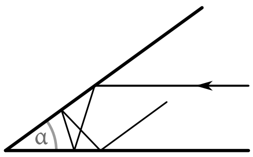
16 Marcelo ya tiene el carnet de conducir de coche y de avión y ahora quiere el de barco. Pero tiene que empezar desde muy abajo, así que de momento trabaja como operador de grúa en un puerto. Como es bastante avispado, rápidamente se le ha ocurrido un nuevo método para levantar contenedores. Su sistema consiste en un cilindro ligero con un radio de 0,99 m metido en un cilindro ligeramente más grande de radio 1 m. Una vara se une al centro de la parte inferior del cilindro pequeño, pasa por el centro en la parte inferior del cilindro grande y en el otro extremo tiene un gancho que levanta el contenedor. Ahora, Marcelo vierte agua en el espacio entre los cilindros hasta que el contenedor se levanta del suelo. ¿Cuánto agua necesita para levantar un contenedor con una masa de 10 t?

Ignore la viscosidad del agua y cualquier fuga alrededor de la vara. La masa de los cilindros es despreciable.



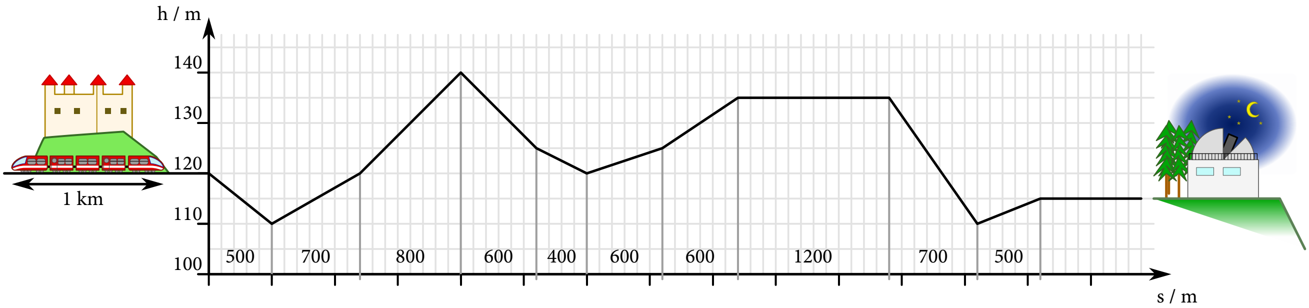
17 Irene ha cogido dos espejos y ha formado con ellos una esquina con un ángulo α. Luego ha apuntado un láser entre ellos, paralelo a uno de los espejos. El haz se refleja entre los espejos y sale de la esquina en la dirección paralela al otro espejo.

¿Cuáles son los posibles ángulos entre los espejos?



18 La electricidad es bastante cara. Para ahorrar, la gestora ferroviaria ordenó a Jacobo, el conductor del ferrocarril, que no acelerase demasiado cuesta arriba y que intentase utilizar el momento lineal del tren lo máximo posible. El tren de Jacobo mide 1000 m y su masa es 1000 t. ¿Cuál es la velocidad mínima a la que tiene que ir el tren desde la estación de la izquierda si quiere cruzar todo el recorrido de la imagen solo con la inercia del tren?

El tren de Jacob tiene una densidad lineal constante. No tengas en cuenta la fricción ni la resistencia del aire. Puedes aproximar $\tan x \approx x$ *para ángulos pequeños.*



19 De nuevo, Sebastián el pistolero está rodeado por forajidos en medio de una plaza en algún lugar del Salvaje Oeste y está planeando una huida espectacular. Se encuentra justo en el centro de un charco circular de whisky muy inflamable cuyo radio es de 5 m, que se ha formado tras una noche de peleas y borracheras.

Ahora solo le quedan tres disparos para encender el combustible. ¿Cúal será el mínimo tiempo necesario para ello? Cada disparo de su revólver puede encender el charco justo en el punto de impacto y las llamas se extienden a un ritmo de 2 m/s.

Sebastián es el pistolero más ágil del Oeste y el tiempo entre disparos y el del vuelo de las balas es despreciable.

20 Un día frío de invierno Daniel encontró dos tiras planas de metal casi idénticas de 1 m de longitud y 1 mm de grosor que alguien había dejado al lado de la basura. La única diferencia entre ambos materiales es su coeficiente de expasión térmica: $8 \cdot 10^{-5}\ \mathrm{K}^{-1}$ y $10^{-4}\ \mathrm{K}^{-1}$. A una temperatura de 0 °C Daniel las colocó una al lado de la otra y las soldó.

¿A qué temperatura esta tira bimetálica tendrá forma de anillo?

Asuma que las tiras solo se expanden a lo largo de su eje más grande.

21 Andrés se ha encontrado tres resistencias: dos son claramente iguales y la tercera es distinta. Usando todas ellas ha construido cinco circuitos eléctricos distintos con resistencias totales de 50, 60, 112, 140 y 245 Ω. Sin embargo, no se ha dado cuenta de que podría haber construido un circuito más.

¿Cúal sería su resistencia?

22 La densidad de flujo de la radiación solar es 1366 W/m^2. Sin embargo, a Julia esto no le hace ninguna gracia: odia que haga demasiado calor en su ático y ha decidido cubrir su ventana con una lámina reflectante especial que sólo deja pasar el 80 % de la luz incidente.

La ventana es cuadrada y tiene un lado de 80 cm. Julia compra un trozo de lámina rectangular de dimensiones 1,5 m × 1 m, la cortó en trozos de forma adecuada y cubrió la ventana con varias capas, aunque eso significara que algunas zonas quedaran cubiertas por más capas que otras. ¿Cuál es la potencia mínima que atravesará ahora la ventana cuando los rayos solares sean perpendiculares al plano de la ventana?

23 Lucía la atleta ha empezado a entrenar para hacer salto de altura. Actualmente es capaz de saltar con suficiente fuerza como para elevar su centro de masa 1 m. Pero eso no es suficiente para ella. Le gustaría mejorar su récord saltando en un pequeño asteroide esférico, que no rota, con un radio de 2,5 km.

¿Cuál es la máxima densidad del asteroide en el que Lucía puede saltar verticalmente y no regresar nunca?

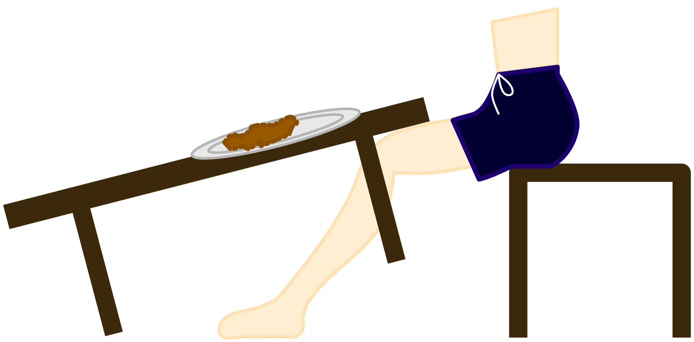
24 Caty, la extraterrestre, recibió financiación de su gobierno, lo que le permitió construir un gran telescopio. Ahora lo utiliza para buscar planetas en las proximidades de otras estrellas. Por ejemplo, si observara nuestro sistema solar, notaría que el Sol se tambalea ligeramente en dirección radial. Un movimiento sinusoidal con un período de un año apuntaría a la existencia de nuestra Tierra, orbitando con el Sol alrededor de un baricentro común.

¿Cuál es la amplitud de la velocidad radial del Sol debido a que la Tierra orbita a su alrededor?

Ignora los efectos de otros planetas. El planeta de Caty, la extraterrestre, se encuentra dentro del plano orbital de la Tierra.

25 Pepe está feliz de que por fin llegue el domingo y tiene un san jacobo con patatas para comer. Lo malo es que sus piernas son tan largas que, una vez que se sienta, la mesa se inclina un ángulo de 30°. El plato con el san jacobo se empieza a deslizar y el san jacobo también se desliza. El coeficiente de fricción entre el plato y la mesa es 0,4, y el coeficiente de fricción entre el plato y el san jacobo es 0,3. El san jacobo tiene masa m y el plato tiene masa $2m$.

¿Cuál es la aceleración del plato justo después de que Pepe se siente?



26 Tomás tomó un muelle de constante k y de longitud en reposo cero y otro un poco más estrecho de constante K y longitud en reposo L. Insertó el muelle pequeño dentro del grande y soldó sus extremos.

¿Cuál es la constante y la longitud sin estirar del objeto resultante?

27 Cuando se quiere determinar la magnitud de una cantidad física, muchas veces es práctico ignorar las constantes numéricas y centrarse en determinar únicamente su dimensión. Estime de esta manera la frecuencia de una cuerda de guitarra vibrando si su masa es M, su longitud es L y la tensión en la cuerda es F.

28 Jorge ha conseguido hacerse con un histograma con el número de terremotos de una determinada magnitud cada año. Los datos se agruparon en intervalos de ancho 1. Jorge dedujo del histograma que hubo 6200 terremotos de magnitud 4,0 – 4,9 y 800 terremotos con magnitudes 5,0 – 5,9. Pero se pregunta cuántos terremotos con magnitud 5,0 ocurrieron.

Jorge sabe que el número de terremotos con una magnitud de al menos M viene dado por la ley de Gutenberg-Richter

$$N(m \geq M) = 10^{a-bM},$$

donde a y b son constantes.

Basándose en las observaciones de Jorge, calcule el número de terremotos que él está tratando de calcular.

La magnitud de un terremoto se redondea a un decimal, lo que significa que el número de terremotos con magnitud 5,0 *en realidad se refiere al número de terremotos con magnitudes exactas en el rango de* 4,95 *a* 5,05. *Del mismo modo, el rango de magnitudes* 4,0 – 4,9 *va de* 3,95 *a* 4,95.

29 Daniel ha notado que cada pocos segundos, el cuarto de la caldera proporciona al sistema de calefacción del edificio 10 l de un líquido misterioso a una temperatura de 80 °C, pero solo 9,9 l de líquido enfriado vuelve al cuarto de la caldera.

¿Cuál es la potencia de la caldera si las tuberías no tienen fugas? El líquido misterioso que hay en las tuberías tiene una densidad de 1000 $\mathrm{kg/m^3}$ en el lugar en el que vuelve a la caldera, un coeficiente volumétrico de expansión térmica de $1{,}8 \cdot 10^{-4}\ \mathrm{K^{-1}}$ y su calor específico es $4000\ \mathrm{J/(kg \cdot K)}$.

30 Cuando Patricio se mudó a su nuevo apartamente trajo consigo un aparato para hacer ejercicio: un muelle duro cuya longitud sin ser estirado es de 1 m. Si lo fija en el techo y se cuelga de él, se estira exactamente 2 m.

Ahora Patricio se pregunta cómo de lejos del techo quedará si fija ambos extremos del muelle en el techo a una distancia de 1 m y se agarra desde el centro del mismo.

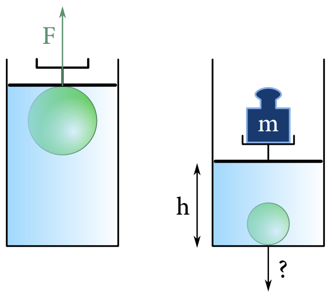
Dé una respuesta con al menos dos cifras significativas. Use la calculadora si lo considera necesario.

31 Andrés y Jacobo están disparándose electrones entre sí. De repente, Andrés se encuentra en una situación bastante delicada, directamente en la trayectoria de un electrón que viaja con velocidad $v = 3200$ m/s. Ahora tiene que recurrir a sus habilidades sobrenaturales – por un breve periodo de tiempo conjura un campo magnético con $B = 8{,}9$ mT en la dirección vertical (perpendicular a v).

¿Cuánto tiempo tiene que mantener el campo magnético para desviar el electrón 90° y que le dé a Jacobo?

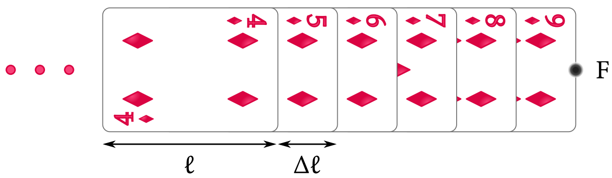
32 Un cilindro recto colocado verticalmente, cuya base tiene un área $S = 50\ \mathrm{cm^2}$, se llena de agua y se cierra con un pistón por arriba. Un globo de volumen $V = 50\ \mathrm{cm^3}$ se mete dentro y flota ejerciendo una fuerza hacia arriba sobre el pistón, $F = 10$ mN. Ahora, una pesa de masa $m = 10$ kg se coloca sobre el pistón y el globo comienza a descender lentamente hasta el fondo. ¿Qué fuerza está ejerciendo sobre el fondo si ahora el pistón se encuentra a una altura $h = 1{,}8$ m desde el fondo?

Asuma que la compresión es isoterma y que las dimensiones del globo son muy pequeñas en comparación con la altura y el radio del cilindro. El cilindro está rodeado de aire a presión atmosférica estándar.



33 Justina está aprendiendo trucos de magia con cartas. Además de practicar trucos de adivinación, también construye castillos de naipes (cartas). Justina acaba de conseguir una baraja de cartas semi infinita, cada carta tiene una masa m y longitud ℓ, cada carta está desplazada por $\Delta\ell$ con respecto a la carta que tiene debajo.

¿Cuál es la fuerza necesaria para levantar la carta de abajo del todo?



34 Cuando Blancanieves no está echándose la siesta, le gusta pasar el rato con los siete enanitos en un tiovivo sin masa. Se sientan en 8 asientos equidistantes alrededor de la circunferencia del tiovivo, girando a una revolución cada tres segundos.

De repente, una manzana cae en el centro del tiovivo. Entonces, Blancanieves se levanta de su sitio y va al centro a recoger la manzana. Tras este suceso, la rotonda gira a una revolución cada 2 segundos.

¿Cuál es la masa de un enanito si Blancanieves pesa 56 kg?

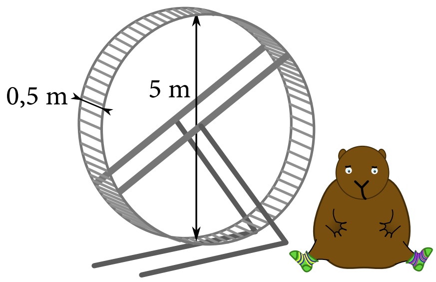
35 Sabrina tiene un cilindro hueco de 10 ml de volumen. El área de la base es de 1 cm^2. El cilindro se llena de aire a presión atmosférica y un disco delgado de 100 g de masa que se mueve libremente por el cilindro lo divide en dos mitades herméticas.

Sabrina no está preparada para manejar un instrumento tan delicado y tras un rato una de las bases del cilindro se salió. ¿Cuál es la proporción entre los periodos de oscilación del disco dentro del cilindro antes y después de que la base se rompiera?

Considere el aire como un gas diatómico ideal. Sabrina se encuentra en la atmósfera estándar.

36 Un hámster demasiado grande está corriendo salvajemente dentro de su rueda. Pronto se supera la capacidad de carga máxima de las bisagras y la rueda se libera. El hámster se cae fuera de la rueda y la rueda cae al suelo y acerela hasta alcanzar una velocidad estable de 1 m/s. ¿Cuál era la velocidad a la que corría el hámster si partió desde el reposo con respecto al cuarto?

La rueda está hecha de varillas de metal con densidad lineal λ. Su circunferencia está formada por un par de anillos de 5 m de diámetro que están conectados por 50 varillas uniformemente espaciadas de 0,5 m de largo. Cada aro está conectado al eje sobre el que rota la rueda mediante una vara cuya longitud es igual al diámetro del aro, y la vara cruza perpendicularmente el eje.



37 Un satélite de masa m está orbitando el Sol con masa M en una órbita circular de radio R. En un punto, activa su motor que empieza a empujarlo con una fuerza constante F en dirección radialmente opuesta al Sol.

¿Cuál debe ser la fuerza del motor para que el satélite alcance una distancia máxima de $2R$ desde el Sol?

38 El oro es extremadamente maleable y se puede martillear hasta formar láminas muy delgadas. Además, es químicamente estable y refleja la luz muy bien. Patricio el Conquistador Cósmico acaba de llegar a una recién descubierta estrella con temperatura T, masa M y radio R. Como saludo para futuras civilizaciones quiere dejar ahí una carta inscrita en una fina placa de oro puro con masa m y densidad ρ.

¿Qué densidad superficial debe de tener la lámina para que la presión de la radiación de la estrella la mantenga a una distancia fija $D \gg R$ de la estrella?

Asuma que la lámina refleja toda la luz y que los rayos de luz incidente son siempre perpendiculares a su superficie.

39 La galaxia Andrómeda está a 2 600 000 años luz de nosotros. ¿A qué velocidad tendríamos que viajar para que la duración del viaje fuese de ese número de años desde nuestra perspectiva?

40 A pesar de los estereotipos, incluso en las grandes ciudades se puede encontrar a gente amable y simpática. Pero también encontramos a Paco. A Paco no le cae bien su vecina de arriba porque no enciende la calefacción ni en invierno cuando hace −5 °C fuera. El techo de Paco tiene un grosor de 20 cm y está hecho de hormigón con una conductividad térmica de 20 W/(m · K). Encima del techo está el suelo de madera de la vecina, con un grosor de 1 cm y una conductividad térmica de 2 W/(m · K).

La temperatura en el piso de Paco se mantiene en 25 °C, lo que hace que en el piso de su vecina haga 10 °C. ¿Cuál será la temperatura en el piso de su vecina después de que Paco instale una capa de espuma de poliestireno de 5 cm de grosor (con conductividad térmica de $1\ \mathrm{W/(m \cdot K)}$) en su techo, con una superficie de $10\ \mathrm{m^2}$?

A Paco le gusta mantener su piso a una temperatura constante. Aparte del suelo, el piso de la vecina solo tiene paredes exteriores.

Soluciones

1 The picture below shows the order in which the tanks are filled. Due to gravity, the water flows down and fills the lowest tank numbered 1. Then the water moves up and displaces the air. The water level in the tanks as well as in the pipes rises, and we fill tanks 2 and 3. Then the triangular tank number 4 fills up, followed by the tank 5. As the water level rises further, the water fills the branch with tanks 6, 7 and 8, and finally the starred tank is filled as ninth.

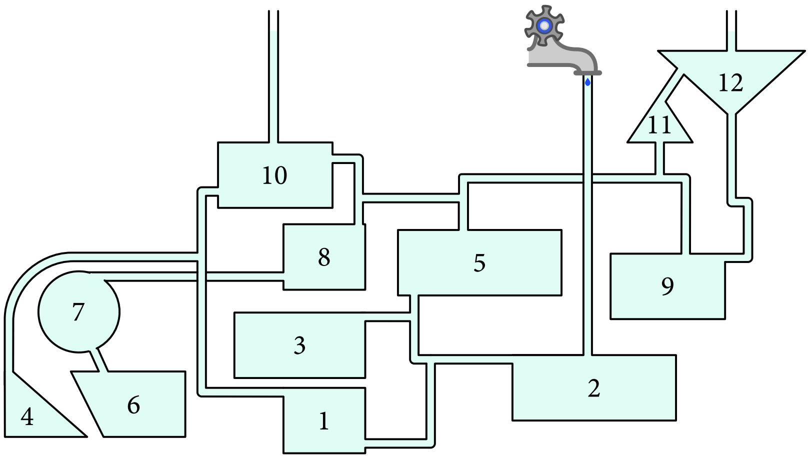


Figura 1.1: *Tank filling order*

2 If one hair grows 15 cm/y, it will grow $\frac{15}{365}$ cm in one day. To get the answer, we simply multiply this by the total number of hairs and convert to metres to obtain $100000 \cdot \frac{15}{365}$ cm/d $\doteq 41$ m/d.

3 If we travel 200 miles per hour, which is $200 \cdot 1760 = 352\,000$ yards per hour, in one second we move $\frac{352000}{3600} = \frac{880}{9}$ yards. The commentator said that this distance is one football field and he is wrong by $120 - \frac{880}{9} = \frac{200}{9}$ yards. In units of football fields this is simply $\frac{200}{9 \cdot 120} = \frac{5}{27}$ of a football field per second.

4 The density is $\rho = \frac{m}{V}$. If the container weighs m, the mass of the biscuits m_b (without the container m_0) is $m_b = m - m_0$. The volume of the container is the difference between the volumes of the cylinder and the raised circular part of its base,

$$V = H\pi R^2 - h\pi r^2. \tag{4.1}$$

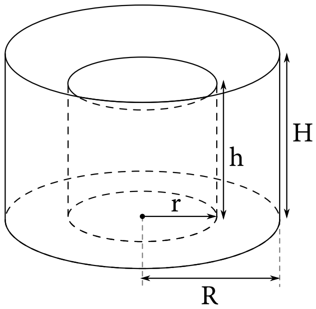


Figura 4.1: *The container with dimensions shown*

The density of the biscuits is therefore

$$\rho = \frac{m_b}{V} = \frac{m - m_0}{H\pi R^2 - h\pi r^2} \doteq 73\ \mathrm{kg/m^3}. \tag{4.2}$$

5 Firstly, let us convert George's velocity to metres per second:

$$\frac{17\ \mathrm{km}}{1\ \mathrm{h}} = \frac{17\,000\ \mathrm{m}}{3600\ \mathrm{s}} = \frac{170}{36}\ \mathrm{m/s}. \tag{5.1}$$

When he steps on the treadmill going at the speed $\frac{180}{36}$ m/s, the difference of the speeds is $\frac{10}{36}$ m/s.

The time needed for him to fall off the treadmill is the time it takes him to cover 1 m at the speed of $\frac{10}{36}$ m/s, so

$$\frac{1\ \mathrm{m}}{\frac{10}{36}\ \mathrm{m/s}} = 3{,}6\ \mathrm{s}. \tag{5.2}$$

6 The whole puddle will be engulfed in flames, when fire reaches the furthermost point. This point lies on the circumference of the puddle and we immediately see some candidates.

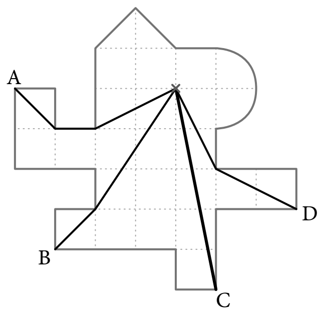


Figura 6.1: *Candidates for the furthermost point*

Using the Pythagorean theorem, we calculate that the furthermost point is C located $\sqrt{26}$ m from the point of ingition. Fire spreads at speed of 2 m/s, therefore whole puddle will be in flames $\frac{\sqrt{26}\ \mathrm{m}}{2\ \mathrm{m/s}} \doteq 2{,}55$ s after ignition.

7 If we suppose that the runner and the car start at the same time, but the car's speed is 0 km/h the first half-hour, then it is enough to determine the time it takes the car to travel 92,14 km. In the first 9 half-hours it travels

$$0\ \mathrm{km} + 7\ \mathrm{km} + 7{,}5\ \mathrm{km} + 8\ \mathrm{km} + 8{,}5\ \mathrm{km} + 9\ \mathrm{km} + 11\ \mathrm{km} + 13\ \mathrm{km} + 15\ \mathrm{km} = 79\ \mathrm{km}. \tag{7.1}$$

After reaching this distance the car is driving at 34 km/h and it still needs to travel 13,14 km. The total running time is then

$$9 \cdot 0{,}5\ \mathrm{h} + \frac{13{,}14\ \mathrm{km}}{34\ \mathrm{km/h}} \doteq 4{,}89\ \mathrm{h}. \tag{7.2}$$

8 Since the numbers are all different powers of two, you have to determine the truthfulness of all statements.

***Common circuit breakers in an apartment have a rated current nominal of about* 150 A.**
The statement is **false**. Standard values are around 16 A. For the voltage in a mains socket 230 V it means a power limit of 3500 W, so four hoovers vacuuming simultaneously would trip the breaker. Breakers at 150 A would easily handle 20 hoovers, which disagrees the common experience.

Galvanic cells are sources of direct current.
The statement is **true**. Galvanic cells, such as a common AA battery, produce electric energy by electrolytic decomposition, which is a process with a given cathode and anode. Therefore, direct current is produced.

Electric forces can occur between insulators.
The statement is **true**. A good example is an electroscope, a basic demonstration of electrostatic force, where two charged plastic balls repel. Therefore, even insulators can be charged on the surface. Have you ever rubbed a balloon against your hair?

Resistivity of electric conductors decreases when they heat up.
The statement is **false**. You can easily search up values of resistivity at different temperatures for any type of conductor in literature. For the majority of common materials, their resistivity increases with rising temperature.

The coulomb is dimensionally equivalent to ampere per second.
The statement is **false**. Electric current, measured in amperes, tells us how much charge (coulombs) passes through per unit time, therefore it's the other way around – amperes are coulombs per second.

Currents smaller than* 1 A *are not dangerous to humans.
The statement is **false**. Even currents as small as 20 mA can be dangerous to humans, especially if passing through one's heart and internal organs. This isn't necessarily related to the current passing through the circuit before we touch it, though. The resistivity in the circuit can be small, therefore huge current can be passing through it even at small voltage. If the resistivity of a human body is large, a small amount of current would pass through it from that small voltage. Conversely, if the voltage is great, but the source cannot produce a large current, we're once again in the clear. A typical example is spark discharge, which can even have 10 000 V, but the transferred charge, and therefore also current, are small.[1]

Two resistors connected in parallel can experience different voltages.
The statement is **false**. Two resistors in parallel are have their ends connected. We know that two points connected by a wire have zero electric potential between them, so the ends of the two resistors have the same voltages. The voltages across both resistors will be the same. A real-life example would be a lightbulb. We often use multiple lightbulbs, with the intention of having each at a 230 V voltage – and this is why they're always in parallel. Even electric sockets are connected in parallel; if we connected two lightbulbs in series, each of them would have half the voltage from the socket, and they would shine at a lower power.

[1] A common question goes: who's the killer, amperes or volts? Do you know?

Even though mains sockets provide alternating current, electric kettles would work with direct current as well.

The statement is **true**. An electric kettle is just a jug with a resistive spiral connected to a bimetallic thermal switch… and in a simpler model, it is just a switch and a resistor that heats up and transfers the heat to the water. Resistors heat up under both direct and alternating currents, so a kettle would work just the same even when connected to a source of direct current.

The sum of the numbers of all true statements is therefore 2 + 4 + 128 = 134.

9 If 5 l of colour weigh 7,5 kg then the density of colour is $\frac{7{,}5\ \text{kg}}{5\ \text{l}} = 1{,}5\ \text{kg/l}$. The density of water is 1 kg/l, which means that the ratio of the densities of the colour and water is 3 : 2. Therefore, if we pour one mass unit of the colour and one of water into two separate containers, their volumes will be in a ratio of 2 : 3. Now we add one more mass unit of colour, which yields a mass ratio of 2 : 1. By doing so, the volume ratio changes to 4 : 3, what is our final result.

10 Granny Justine has to pour mass m of tap water in a jug in which she then puts a block of ice of mass $m_{❄}$. In order to melt the whole icy block, it has to absorb heat equal to $m_{❄} l_{❄}$, where $l_{❄}$ is specific latent heat of fusion of water. Then, the melted ice, also known as water, heats by $\Delta T_{❄}$, so it absorbs heat $m_{❄} c_{H_2O}\, \Delta T_{❄}$, where c_{H_2O} is specific heat capacity of water. All this heat is absorbed from the tap water, which cools by ΔT_{H_2O} so it gives away heat of $m c_{H_2O}\, \Delta T_{H_2O}$ to the ice.

Heat absorbed by the ice is equal to heat given away by the tap water, thus

$$m_{❄} l_{❄} + m_{❄} c_{H_2O}\, \Delta T_{❄} = m c_{H_2O}\, \Delta T_{H_2O}. \tag{10.1}$$

We solve for mass of the tap water

$$m = \frac{m_{❄} l_{❄} + m_{❄} c_{H_2O}\, \Delta T_{❄}}{c_{H_2O}\, \Delta T_{H_2O}}. \tag{10.2}$$

The values from the problem statement give $m \doteq 6{,}68$ kg. Apart from tap water, the jug contains also 1 kg of water which stems from the ice, so, all in all, there is 7,68 kg of water in the jug which is 7,68 l.

11 Let t_0 denote the time of Sarah's scream. The main idea here is that Sarah's sound, which propagates at a constant velocity c, will overtake linearly accelerating Matt at some time t at which the distances traversed by both Matt and the sound of the scream will be the same

$$\begin{aligned} \frac{1}{2} g t^2 &= c(t - t_0) \\ \frac{1}{2} g t^2 - ct + ct_0 &= 0. \end{aligned} \tag{11.1}$$

This is a quadratic equation with two roots

$$t_{\pm} = \frac{c \pm \sqrt{c^2 - 2gct_0}}{g}. \tag{11.2}$$

The numerical values of these roots are $t_+ \approx 66{,}79$ s and $t_- \approx 3{,}14$ s respectively. Physically, these correspond to when the sound first overtakes Matt and, much later, when constantly accelerating Matt overtakes the sound again (since we're disregarding air friction). Hence $t = t_-$.

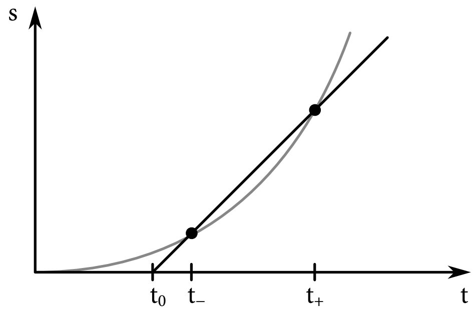


Figura 11.1: *Distances travelled by Matt and Sarah's scream*

Now that we know time t, we can substitute it into the equation of linearly accelerated motion

$$s = \frac{1}{2} g t_-^2 \approx 48{,}4 \text{ m}. \tag{11.3}$$

Matt jumped from the height $h = 200$ m, therefore at time t when he hears Sarah's roar he will be at height of $h - s \approx 151{,}6$ m.

12 Let us denote the value of the resistor's resistance R. If we have a series connection, we sum the resistances of the resistors, so the total resistance will be $3R$. In parallel connection, we are summing reciprocal values of the resistances, so the resistance is equal to $R/3$. These values differ by 8 Ω, so we can construct the equation

$$8\ \Omega = 3R - \frac{R}{3}. \tag{12.1}$$

The solution of this equation is $R = 3\ \Omega$. The diagram in the figure in the problem statement is a parallel connection of two branches with resistances R and $2R$, so its total resistance can be calculated as

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R} \quad \Rightarrow \quad R' = 2\ \Omega. \tag{12.2}$$

13 Francis uses the same amount of kinetic energy for his jump and push off the wall. In the highest point of his jumps, all kinetic energy will change to potential energy which will be the same in both cases. It means that the height of the jump will be equal. The height can be determined from the picture geometry as $h = \ell - \sqrt{\ell^2 - d^2} = 1{,}2$ m.

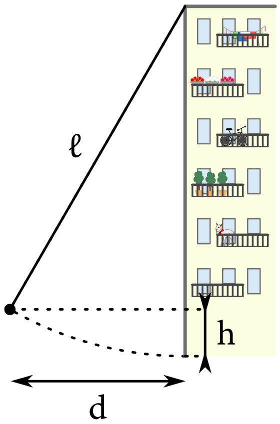


Figura 13.1: *Geometry of Francis' jump*

14 If the wheel rotates, the tyre valve rotates with it. It means that in the frame reference of the rotating wheel with the origin in the centre of the tyre the valve stands still. The fictitious centrifugal force acting on the valve is $F = \frac{mv^2}{r}$ which is oriented from the origin. However, the valve is not moving, so there must be opposite force with the same magnitude from the wheel. When the car accelerates, in the frame reference of the wheel it seems that the valve changes its weight, however the only thing that changes is the frequency of rotating wheel. The apparent weight can be computed as

$$m_a = \frac{F}{g} = \frac{mv^2}{rg}.$$

After the unit conversion from km/h to m/s and using the given values we get $m_a \approx 98{,}3$ kg.

15 It is necessary to realize that if Julia has only one cover, she will cover the greatest area of the pool if she places it into the corner of the pool so that its centre protrudes over the water as much as possible. This situation is depicted in the figure below. The centre of mass must be located on the line segment connecting the two points that are the intersections of the cover's circumference with the perimeter of the pool. In this case, the corner of the pool also lies on the circumference of the cover.[2] If Julia moved the cover a little more over the water, it would fall into the pool. Therefore it is enough to calculate the area of water that is covered. This area consists of a semicircle and a right triangle. The area of a semicircle is simply $\frac{1}{2}\pi r^2$.

[2] thanks to Thales's theorem

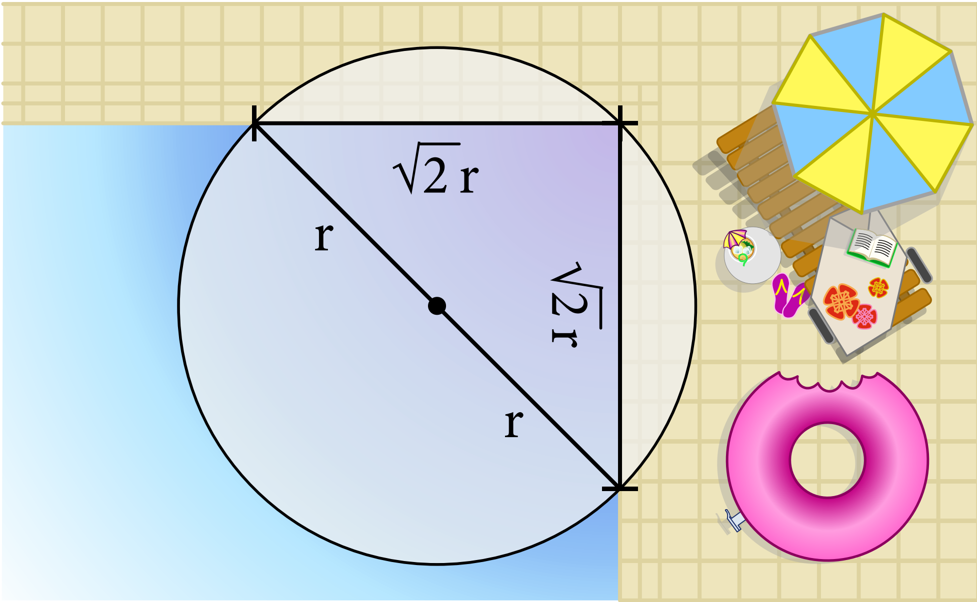


Figura 15.1: *The cover in optimal location*

We know that the right triangle's hypotenuse has a length of $2r$ and its other two sides have a length of $\sqrt{2}r$ because the Pythagoras' theorem must hold: $\sqrt{\left(\sqrt{2}r\right)^2 + \left(\sqrt{2}r\right)^2} = 2r$. The area of this triangle is $\frac{1}{2}(\sqrt{2}r)^2 = r^2$. The total area covered by the circle is then

$$\frac{1}{2}\pi r^2 + r^2. \tag{15.1}$$

16 Let us draw and write down all the forces that act on the axis of the cylinder:

$$F_g + F = F_{\mathrm{vz}}, \tag{16.1}$$

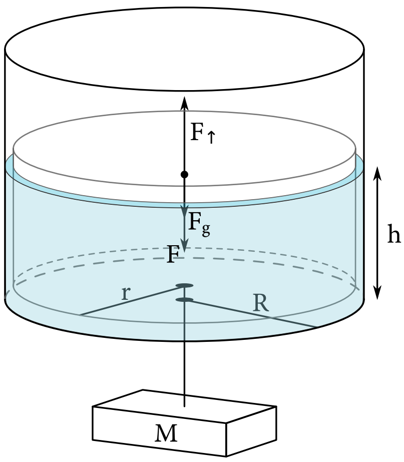


Figura 16.1: *Marcel's crane with forces*

where F_g is the gravitational force, F is the force from the container, and F_{vz} is the buoyant force of the water. After expressing the individual forces, we get

$$mg + Mg = \rho_{H_2O} g h S_1, \tag{16.2}$$

where h is the height of the submerged part of the cylinder and S_1 is the base of the smaller cylinder. To find the volume, we need to find h,

$$h = \frac{m + M}{\rho_{H_2O} S_1}. \tag{16.3}$$

We know that the mass of the smaller cylinder is negligible compared to the container, so

$$h \approx \frac{M}{\rho_{H_2O} S_1}. \tag{16.4}$$

The volume of water will eventually be just the volume difference of the cylinders, so

$$V = h(S_2 - S_1) = \frac{M}{\rho_{H_2O} r^2}(R^2 - r^2) \doteq 203\,\mathrm{l}. \tag{16.5}$$

17 Instead of calculating complicated geometry, we realise that we just need to mirror the space between the mirrors a few times. Then we can consider that a ray of light travels along a straight trajectory as in the picture.

At the end, the ray of light must be parallel to the other mirror. This means that the angle 180° must be divided into a whole number of parts by the angles α. Thus, it would seem that the valid values of α would be $\frac{180°}{n}$, where n are natural numbers. However, for the ray at the end to be parallel to the other mirror it is necessary, that the mirror which appears at an angle 180° is the other mirror.

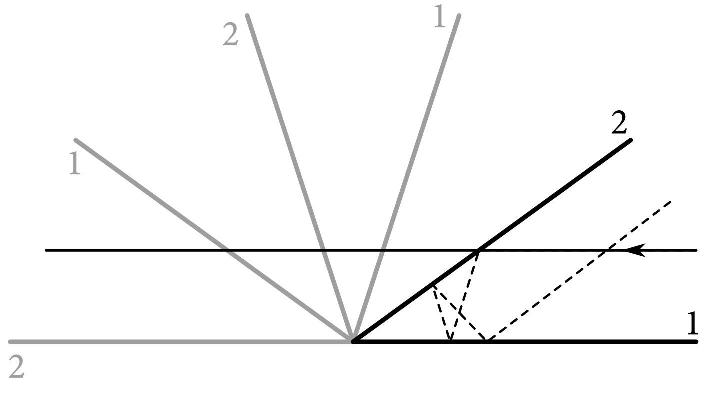


Figura 17.1: *Solution with just four reflections*

We quickly realise that this condition tells us that the number n must be odd. Therefore the valid values of the angle α are

$$\alpha = \frac{180°}{2n + 1}, \tag{17.1}$$

where $n \in \mathbb{N}$.

18 The train starts off with speed v and thus has a certain amount of kinetic energy. Since neither friction, nor air resistance play a role, total mechanical energy of the train is conserved. Therefore it only needs to travel at such a speed so that its centre of mass just passes over the highest point of its trajectory. However, since the train is not a point mass, the trajectory of its centre of mass differs from the altitude profile of the track. At a glance, we can determine that two areas come into consideration as highest: the peak at 140 m and the plateau at 135 m. Obviously, there is no way or place to get any higher.

When we calculate the slopes, we can see that they are all quite small, the steepest drop is only $\frac{25}{700}$. For small angles we can neglect the difference between the actual length of the train and its projection onto the horizontal plane, which will simplify the calculation somewhat. Then we can also ignore the height of the train's centre of gravity above the rails, as this will always be the same.

Let us first look at the highest point of the track. The slopes on both sides are equal, 25 ‰, so the train's centre of gravity will be at the highest point when it passes over the summit. Since the linear density of the train is constant, the centres of gravity of both the front and rear halves of the train are at equal altitudes, and hence the centre of gravity of the whole train is at that altitude as well. So we just need to calculate the height of the centre of gravity of one of the halves, which will be

$$h_\wedge \approx \frac{140\ \mathrm{m} + (140\ \mathrm{m} - 0{,}025 \cdot 500\ \mathrm{m})}{2} = 133{,}75\ \mathrm{m}. \tag{18.1}$$

However, this is obviously less than the height of the straight part on the right, where the train fits entirely. The height of its centre of gravity on it will therefore be trivially $h_- = 135$ m.

All that remains is to express the difference in the heights of the centre of gravity at the start and at the maximum height, which is

$$\frac{1}{2}mv^2 = mgh_- \quad \Rightarrow \quad v = \sqrt{2gh_-} \approx \sqrt{2 \cdot 10\ \mathrm{m/s^2} \cdot 15\ \mathrm{m}} = \sqrt{300}\ \mathrm{m/s} \doteq 17{,}3\ \mathrm{m/s}. \tag{18.2}$$

19 Let us denote v the speed at which fire spreads, and t the time when the whole puddle is burning. Our task is to cover the puddle with three circles with radius vt and optimalise their placement to minimise t. We divide the puddle into three sections, each being inflamed by one of the pistolero's bullets. We want these sections to be as small as possible, which means we want to make the circle covering it as small as possible. Symmetry of the puddle allows us to divide it into three equally sized circular sectors and a simple thought convinces us that it is not possible to find a better solution. Ideal points of bullet impacts can be found as centres of the circles.

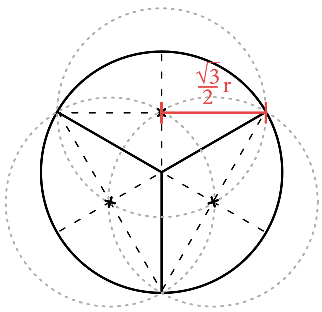


Figura 19.1: *Anatomy of Sebastian's inflammatory activity*

A short exercise from geometry reveals us that the longest distance which has to be travelled by flame has length of $\frac{\sqrt{3}}{2}r$, where r is radius of the puddle. Entire puddle will therefore be engulfed in flames in time $\frac{\sqrt{3}}{2}\frac{r}{v} = \frac{5\sqrt{3}}{4}$ s $\doteq$ 2,17 s.

20 Since the thickness of the strips is one thousand times smaller than their length, we can neglect the changes in their thickness due to temperature changes and assume that only their length varies. Let r_1 and r_2 be the radius of the inner and outer strip, respectively. If we denote their thickness h, it then follows $r_2 = r_1 + h$. The strip with original length l_0 and thermal expansion coefficient α will, due to a temperature difference ΔT, increase its length by $l_0\alpha\,\Delta T$. Its new length will therefore be

$$l_0 + l_0\alpha\,\Delta T = l_0(1 + \alpha\,\Delta T). \tag{20.1}$$

If the strip has a circular shape with radius r, its length is equal to $2\pi r$. For Daniel's newly acquired strips this can be expressed as

$$\begin{aligned} l_0(1 + \alpha_1\,\Delta T) &= 2\pi r_1, \\ l_0(1 + \alpha_2\,\Delta T) &= 2\pi r_1 + 2\pi h, \end{aligned} \tag{20.2}$$

using $r_2 = r_1 + h$. We can subtract the second equation from the first and we get

$$\Delta T = \frac{2\pi h}{l_0(\alpha_2 - \alpha_1)}. \tag{20.3}$$

This means we have to either cool down or warm up the two strips by ΔT. For given values $\Delta T \doteq 314$ K. Since the dimensions of the strips were measured at 0 °C $\doteq$ 273 K, we have to increase the temperature by ΔT, since it is impossible to achieve negative thermodynamic temperature. The bimetallic strip will take circular shape at the temperature 314 °C.

21 When solving this problem we will not avoid guessing. This does not mean that we can't guess wisely. In total, there are six possible circuits. We will guess that the purely parallel and the purely serial wiring has the smallest and the largest resistance, respectively. These have following resistances

$$\frac{R_1R_2}{R_1 + 2R_2} \qquad \text{and} \qquad 2R_1 + R_2, \tag{21.1}$$

where R_1 is the resistance of two identical resistors and R_2 is the resistance of the third resistor.

But if we consider that the smallest and the largest resistance is 50 Ω and 245 Ω, we get a system of two equations that has no solution. This means that the value we are missing must be just the smallest or the largest. So we need to guess which circuit corresponds to the second smallest, or the second largest resistance, respectively.

Firstly, we can notice that the resistors in parallel always have lower resistance, than the individual resistors, and when connected in series higher resistance. In the case of two parallel branches, the resulting resistance is always lower than the resistances in the individual branches. In the case of a series connection, the resulting resistance is higher than the individual resistors connected in series. This allows us to assume that the smaller resistances correspond to parallel wiring and larger resistances to serial wiring, reducing the number of possibilities we need to explore.

Next, we note that since parallel wiring reduces resistance, it is not worth including small resistors, when maximalizing resistance. Thus, if two identical resistors have less resistance than the third one, it can be assumed that the parallel connection of two identical resistors and the third in series with them will have the second largest resistance.

In addition, we can note that if one branch of the parallel wiring has significantly more resistance than the other, the current will mostly flow through the branch with lesser resistance, which puts up little resistance. In the limiting case, when the resistance of this branch goes to zero, current can flow through the circuit with virtually no resistance and no current flows through the other branch. On this basis, it can be concluded that if two equal resistors have more resistance than the third one, a series connection of a pair of identical resistors in one branch and to them in parallel a smaller resistance will have the second smallest resistance.

This reduced the number of cases we have to examine to four:

- If the smallest resistance is missing, the largest resistance corresponds to the series circuit of all three resistors. Then, the missing resistance corresponds to the parallel circuit of all three resistors.
  - If the same resistors have lower resistance than the third, the two identical resistors in parallel and the third in series corresponds to the second largest resistance.
  - If the same resistors have resistance greater than the third, the two identical resistors in series in one branch and a third in parallel to them corresponds to the smallest known resistance[3].
- In the absence of the largest resistance, the smallest resistance corresponds to the parallel wiring of all three resistors. Then the missing resistance corresponds to the three resistors in series.
  - If the same resistors have smaller resistance than the third, two identical resistors in parallel and the third in series corresponds to the largest known resistance[4].
  - If the identical resistors have greater resistance than the third, two identical resistors in series on one branch and the third in parallel to them corresponds to the second smallest resistance.

Let us explore these possibilities. Doing so, we may notice that the largest resistance on the list is significantly larger than the next two resistances on the list. Therefore, we may assume that the largest resistance corresponds to the serial connection of all three resistors, and that the next two resistances correspond to the circuits in which one resistor is serially connected to a couple of two resistors conected in parallel. Thus, let us start with the first two options.

We already know the resistance of all three resistors in series. The resistance of identical resistors R_1 in parallel and R_2 in series has a resistance $\frac{R_1}{2} + R_2$. Solving this system of two equations gives the resistances $R_1 = 70\ \Omega$ and $R_2 = 105\ \Omega$. We can easily verify that the remaining resistances from the problem correspond to one of the circuits with such resistors. We do not need to investigate other possibilities – we have already found the solution.

The missing resistance corresponds to all three resistors in parallel, i.e. 26,25 Ω.

22 First we have to decide which strategy to use when covering the window with foil. Apparently, we will be able to cover the entire window twice and still have some foil left over. Using the remaining foil, we will

[3] The resistance of a parallel connection of all three resistors, which certainly has a smaller resistance, is unknown

[4] Resistance of series wiring of all three resistors, which certainly has the greater resistance, is unknown

achieve the greatest filtering effect if we cover that area of the window surface which has the smallest number of layers covering it so far. That means, we will try to cover the entire window as uniformly as possible.

The surface of the foil is 1,5 m^2, the surface of the window is $S = 0{,}64\ \mathrm{m}^2$. To cover the entire window twice, we will use 1,28 m^2 of foil and will have $0{,}22\ \mathrm{m}^2 =: S_3$ of foil left over. That means that the surface S_3 will be covered three times and the surface $S_2 = S - S_3 = 0{,}42\ \mathrm{m}^2$ will be covered twice.

How much power will pass through multiple layers of foil can be easily calculated. Each layer will allow 80 % of incident light to pass through. It then follows that two layers will allow $\alpha_2 = 80\ \% \cdot 80\ \% = 64\ \%$ and three layers of foil will allow $\alpha_3 = 80\ \% \cdot 64\ \% = 51{,}2\ \%$ of light to pass through.

Knowing the area covered by two and three layers respectively, the total power passing through the entire window can be calculated as

$$P' = (\alpha_2 S_2 + \alpha_3 S_3)P = \left(0{,}64 \cdot 0{,}42\ \mathrm{m}^2 + 0{,}512 \cdot 0{,}22\ \mathrm{m}^2\right) \cdot 1366\ \mathrm{W/m^2} \doteq 521\ \mathrm{W}. \tag{22.1}$$

23 Let Lucy have zero potential energy and some kinetic energy at the moment she leaves the ground. At the highest point of the jump, at height h, she stops momentarily, so she only has potential energy mgh, where m is Lucy's mass. This means that her kinetic energy at the start of the jump is equal to mgh, because the law of conservation of mechanical energy holds during the jump.

If Lucy wants to jump off the asteroid, she has to reach the infinity. Here, we can no longer use the relation for the potential energy in a homogeneous gravitational field as we did for the jump on the Earth. Instead, we have to take into account the radial field of the asteroid. In the limiting case, when the asteroid has the largest possible mass for Lucy to jump off, Lucy arrives at infinity and stops there after an infinitely long time.

Lucy's potential energy at the surface of the asteroid is $-\frac{GMm}{R}$, where M is the mass of the asteroid and R is its radius. At the start of the jump she gains kinetic energy equal to mgh. At infinity, Lucy stops moving and her potential energy is zero. Due to the conservation of energy, the sum of the kinetic and potential energy at the moment of lift-off is equal to the sum of the kinetic and potential energy at infinity,

$$mgh - \frac{GMm}{R} = 0 + 0. \tag{23.1}$$

From here we can express the mass of the asteroid in the limiting case,

$$M = \frac{Rgh}{G}, \tag{23.2}$$

and then the density as

$$\rho = \frac{M}{V} = \frac{3gh}{4\pi R^2 G} \approx 5618\ \mathrm{kg/m^3}.$$

24 Let us denote the mass of the Sun as $M_\odot$ and the mass of the Earth as $M_\oplus$. Their mutual distance is R. Both of these objects revolve around the common centre of mass with some angular velocity. First, we will need to find the centre of mass. Let us denote the distance between the Sun and the centre of mass as r. Then

$$M_\odot r = M_\oplus (R - r) \tag{24.1}$$

which yields

$$r = \frac{M_\oplus}{M_\odot + M_\oplus} R. \tag{24.2}$$

The Sun revolves around the centre of mass on a circular orbit with radius r and speed v. After realising that this is caused by the gravitational force we get

$$M_\odot \frac{v^2}{r} = G \frac{M_\odot M_\oplus}{R^2}. \tag{24.3}$$

Now it is enough to plug in the radius r from the equation 24.2 and to express v. We obtain

$$v = \sqrt{G \frac{M_\oplus^2}{R(M_\odot + M_\oplus)}}, \tag{24.4}$$

which after evaluation yields $v \approx 9$ cm/s.

When the Sun moves perpendicular to the line connecting the barycentre and the alien observer, its radial velocity is zero. If it moves towards of away from the observer, its radial velocity is $\pm v$. Therefore v is exactly the amplitude of the radial velocity of the Sun.

25 In plotting the forces acting on the plate and the schnitzel, we come across the main non-trivial idea of the problem. What is the relative acceleration of the plate and the schnitzel? In fact, the schnitzel has a greater or equal acceleration than the plate, a reason we will now discuss. Consider the situation where f_1, the coefficient of shear friction between the plate and the table, is zero. In that case, the plate and the schnitzel will move with acceleration $g \sin \alpha$. Thus, their relative acceleration will be zero and the schnitzel will not move relative to the plate.

Let's put this situation in the horizontal direction. Nothing will move. If we add a force to represent the frictional shear force between the plate and the table (F_t), that is the force that acts only on the plate, we can easily see that the schnitzel will want to stay in place relative to the table. Therefore, the plate (T) will exert a frictional force on the schnitzel, which will prevent it from staying still. T will cause no more acceleration than F_t causes. We can see that the schnitzel will go down faster than the plate.

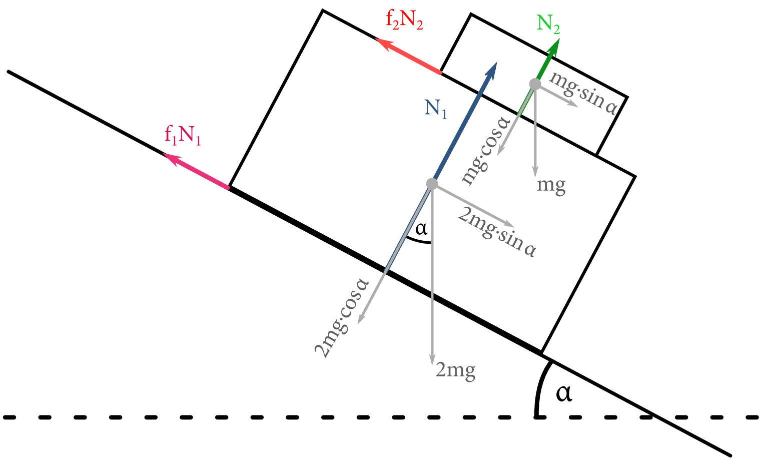


Figura 25.1: *Sketch of the forces acting on the plate and the schnitzel.*

From the image we can see all the forces and we can write the equations of motion for the plate,

$$2ma = 2mg\sin\alpha - (2m+m)g f_1\cos\alpha + mg f_2\cos\alpha, \tag{25.1}$$

where f_2 is the coefficient of shear friction between the plate and the schnitzel.

The first term on the right-hand side of the equation 25.1 represents the component of the gravitational force that moves the plate down the table, the second represents the shear frictional force between the plate and the table (for a mass of $3m$, because the top of the plate also pushes on the schnitzel) and the third term is the reaction force to the shear friction force between the schnitzel and the plate.

After adding the values and expressing a, we get the resulting acceleration of the plate $a \approx 1{,}08\ \mathrm{m/s^2}$.

26 Let us denote the total length of the resulting spring as ℓ. The first spring is stretched by force $k\ell$ and the force with with the second spring is compressed is $K(L-\ell)$. These forces must be equal, which yields

$$k\ell = K(L-\ell) \quad \Rightarrow \quad \ell = \frac{KL}{k+K}. \tag{26.1}$$

Now imagine that this system of springs is stretched by a distance $\Delta\ell$ so that its total length is $\ell + \Delta\ell$. What is the total force? First spring will be stretched with a force $k(\ell+\Delta\ell)$ and the second will be compressed by a force $K(L-\ell-\Delta\ell)$. These two forces act in opposite directions, so we need to subtract them. The resulting force is

$$F = k(\ell+\Delta\ell) - K(L-\ell-\Delta\ell). \tag{26.2}$$

Finally, we use the equation 26.1, which simplifies the previous equation to

$$F = (K+k)\,\Delta\ell \tag{26.3}$$

from which it is easy to see that the total stiffness is $K+k$.

27 Obviously, we will use physical quantities given in the problem statement. Let the guitar string frequency be

$$f \sim F^a M^b L^c \text{ for some } a, b, c \in \mathbb{R}. \tag{27.1}$$

We did not write the equal sign because we are not interested in numerical constants. Those are not determinable by dimensional analysis. Values of a, b, c are to be determined so that the product of physical quantities in 27.1 has dimension of frequency, that is $\mathrm{s^{-1}}$. Apart from that, newton expressed in SI base units is $\mathrm{kg \cdot m \cdot s^{-2}}$.

The right side of the 27.1 in the language of dimensions is

$$\left(\mathrm{kg\cdot m\cdot s^{-2}}\right)^a \mathrm{kg}^b \mathrm{m}^c = \mathrm{kg}^{a+b}\mathrm{m}^{a+c}\mathrm{s}^{-2a}, \tag{27.2}$$

which has to be of same dimension as frequency, so

$$\mathrm{s}^{-1} = \mathrm{kg}^{a+b}\mathrm{m}^{a+c}\mathrm{s}^{-2a}. \tag{27.3}$$

In order for this equality to be true, the exponent of each physical quantity has to be same on both sides of the equation. A swift look shows that $a = \frac{1}{2}$, which immediately leads to $b = -\frac{1}{2}$ and $c = -\frac{1}{2}$. Thus, a guitar string vibrates with frequency $f \sim \sqrt{\frac{F}{ML}}$.

Fun fact, if we calculated it properly, the fundamental frequency of a guitar string is $f = \frac{1}{2}\sqrt{\frac{F}{ML}}$.

28 From the problem statement

$$N(m > M) = 10^{a-bM}, \tag{28.1}$$

so for the number of earthquakes in the interval 4,0 – 4,9 we can write

$$\begin{aligned} N(3{,}95 \leq m < 4{,}95) &= N(m \geq 3{,}95) - N(m \geq 4{,}95) \\ &= 10^{a-3{,}95b} - 10^{a-4{,}95b} \\ &= 10^{a} \cdot 10^{-4{,}95b}\left(10^{b} - 1\right) \\ &= \alpha \cdot \beta^{-4{,}95}(\beta - 1), \end{aligned} \tag{28.2}$$

where we introduced the notation $10^a \equiv \alpha$ and $10^b \equiv \beta$. By analogy, for the interval 5,0 – 5,9 we have

$$\begin{aligned} N(4{,}95 \leq m < 5{,}95) &= N(m \geq 4{,}95) - N(m \geq 5{,}95) \\ &= 10^{a-4{,}95b} - 10^{a-5{,}95b} \\ &= 10^{a} \cdot 10^{-5{,}95b}\left(10^{b} - 1\right) \\ &= \alpha \cdot \beta^{-5{,}95}(\beta - 1). \end{aligned} \tag{28.3}$$

For the number of earthquakes with magnitude 5,0,

$$\begin{aligned} N(4{,}95 \leq m < 5{,}05) &= N(m \geq 4{,}95) - N(m \geq 5{,}05) \\ &= 10^{a-4{,}95b} - 10^{a-5{,}05b} \\ &= 10^{a} \cdot 10^{-4{,}95b}\left(1 - 10^{-0{,}1b}\right) \\ &= \alpha \cdot \beta^{-4{,}95}\left(1 - \beta^{-0{,}1}\right). \end{aligned} \tag{28.4}$$

When we divide 28.2 by 28.3, we get

$$\beta = \frac{N(3{,}95 \leq m < 4{,}95)}{N(4{,}95 \leq m < 5{,}95)}. \tag{28.5}$$

Equation 28.2 moreover implies,

$$\alpha \cdot \beta^{-4{,}95} = \frac{N(3{,}95 \leq m < 4{,}95)}{\beta - 1}. \tag{28.6}$$

For the number of earthquakes we are looking for, we get

$$N(4{,}95 \le m < 5{,}05) = N(3{,}95 \le m < 4{,}95) \cdot \frac{1-\beta^{-0{,}1}}{\beta - 1}, \tag{28.7}$$

After evaluation with given values from the problem statement $N(4{,}95 \le m < 5{,}05) \approx 170$.

29 The heating station heats the school with its thermal power. We need to find how many joules per second are transferred from the mysterious liquid in the radiators to the air in the building. If the liquid changes its temperature by ΔT, its volume changes by $V_0 \beta \, \Delta T$, where V_0 is its original volume and β is volumetric coefficient of thermal expansion.[5]

The change of temperature is thus

$$\Delta T = \frac{\Delta V}{V_0 \beta}. \tag{29.1}$$

The heat transferred from the liquid to the building is simply

$$Q = mc \, \Delta T, \tag{29.2}$$

where m is mass of the mysterious liquid and c is its specific heat capacity. The liquid has the same density as water, so the outgoing 9,9 l per second weigh $m = 9{,}9$ kg. After plugging this into 29.2, we find out that $Q \approx 2{,}2$ MJ. Since this heat is dissipated in one second, the power of the heat station is 2,2 MW.

30 Let us first note that if Patrick with mass m is suspended from a spring with a stiffness k, then its extension will be

$$\Delta L = \frac{mg}{k}. \tag{30.1}$$

If Patrick is suspended by the centre of the spring as in the problem, he will be suspended by height d, see figure 30.1. We can split the whole spring in half into two springs of stiffness $2k$. The rest length of each is $\frac{L}{2}$ and the actual length of each is $\sqrt{\left(\frac{L}{2}\right)^2 + d^2}$.

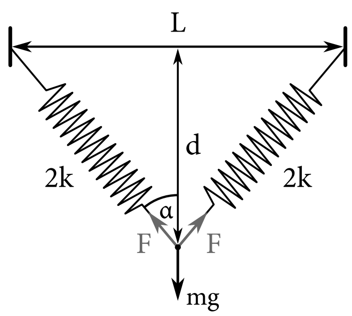


Figura 30.1: *Geometry of a suspended Patrick*

[5]This relationship is in fact exponential, but as the change in temperature is small we can use a linear approximation.

The force F that each of the springs exerts on Patrick is then

$$F = 2k\left(\sqrt{\frac{L^2}{4} + d^2} - \frac{L}{2}\right). \tag{30.2}$$

In order for Patrick to be in equilibrium, the forces from the springs must balance with the gravitational force. Thus, the equation

$$mg = 2F\cos\alpha \tag{30.3}$$

must hold, where we can see from the geometry that

$$\cos\alpha = \frac{d}{\sqrt{\frac{L^2}{4} + d^2}}. \tag{30.4}$$

When we substitute the force F and $\cos\alpha$ to the equations 30.2, 30.3, we get

$$mg = 4k\left(\sqrt{\frac{L^2}{4} + d^2} - \frac{L}{2}\right)\frac{d}{\sqrt{\frac{L^2}{4} + d^2}}. \tag{30.5}$$

We now divide this equation by the stiffness k and use the equation 30.1,

$$\begin{aligned} \Delta L &= 4\left(\sqrt{\frac{L^2}{4} + d^2} - \frac{L}{2}\right)\frac{d}{\sqrt{\frac{L^2}{4}4 + d^2}} \\ -4Ld &= \sqrt{L^2 + 4d^2}(\Delta L - 4d) \end{aligned} \tag{30.6}$$

and we square this to get rid of the square root

$$\left(L^2 + 4d^2\right)(\Delta L - 4d)^2 = 16L^2d^2. \tag{30.7}$$

Now we get a quartic equation

$$L^2\,\Delta L^2 - 8L^2\,\Delta L d + 4\,\Delta L^2 d^2 - 32\,\Delta L d^3 + 64d^4 = 0 \tag{30.8}$$

with an unknown length d. This one is quite difficult to solve analytically, but after plugging in all numerical constants $L = 1$ m and $\Delta L = 2$ m, we can solve it numerically.

The simplest way to find the root of the equation numerically is by binary search. Let us denote the left-hand side of the equation 30.8 as a function $f(d)$. First, we guess some values for d and evaluate $f(d)$. We get

$$f(0) = 4, \quad f(1/2) = -4 \quad \text{and} \quad f(1) = 4. \tag{30.9}$$

From the signs of these results, it follows that one root will lie in the interval 0 – 0,5 m and the other in the interval 0,5 – 1 m. Next, we proceed by dividing these intervals in half and, according to the sign of the function $f(d)$, narrow the interval each time until we reach the desired precision.

We get two roots, $d = 0{,}2655$ m and $d = 0{,}9416$ m. However, after plugging it back into the equation 30.6, we find, that the first of the solutions does not satisfy the equation. Thus, this solution is not physically correct and was generated in step, where we have made a square from equation 30.6. Thus, the height by which Patrick suspends is $d \approx 0{,}94$ m.

31 Electrically charged particles are subject to both electric and magnetic forces. Whilst electric fields simply act repulsively or attractively, analogous to gravity, magnetic fields have a more complex effect, which is described by the equation

$$\vec{F} = q\vec{v} \times \vec{B}. \tag{31.1}$$

Let us describe the situation in cartesian coordinates. Let us say the electron comes in the positive x direction and the magnetic field everywhere is in the negative z direction (a common convention). Then, at the moment the magnetic field is turned on, the force on the electron acts in the positive y direction. As the trajectory of the electron curves towards the positive y direction, the direction of the force rotates simultaneously so that it is always perpendicular to both the instantaneous velocity and the magnetic field.

As the z-component of the electron's velocity is initially zero and the z-component of the force must be always zero, the electron is trapped in the xy plane. And since the force is always perpendicular to the electron's velocity, the velocity's magnitude will stay constant, only its direction changes, and that at a constant rate (since no quantity in the force law changes magnitude). Therefore, the electron moves in a circle.

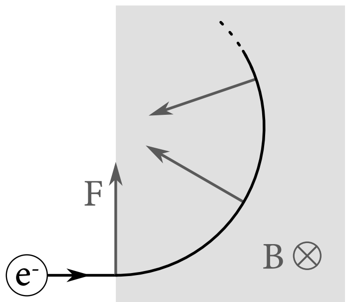


Figura 31.1: *The electron's trajectory*

Since the direction of the motion is always perpendicular to the magnetic field, we can express the magnitude of F by changing the cross product in 31.1 to a simple multiplication of v and B. Since this is the force causing the circular motion, we can equate it with the expression for centripetal acceleration and isolate R,

$$\begin{aligned} m_e\frac{v^2}{R} &= q_e vB, \\ R &= \frac{m_e v}{q_e B}. \end{aligned} \tag{31.2}$$

This is called Larmor radius. We are interested in the time it takes the electron to traverse a quarter of the orbit's circumference, with this segment having the length of $\frac{\pi}{2}R$. This time is then

$$t = \frac{\pi}{2v}R = \frac{\pi}{2v}\frac{m_e v}{q_e B} = \frac{\pi m_e}{2q_e B}. \tag{31.3}$$

We know all the quantities in this formula – elementary charge, electron mass, strength of the magnetic field – hence we obtain $t \approx 1$ ns.

32 If we denote the mass of the balloon as M, then $F = V\rho g - Mg$, where ρ is the density of water. Similarly, the force that the balloon eventually exerts on the bottom is $F' = Mg - V'\rho g$, since water is incompressible and, therefore, does not change its density. The problem states that the compression of the balloon is isothermal, so $pV = p'V'$. It remains for us to find the values of the pressure. Initially the balloon is at atmospheric pressure $p = p_{\mathrm{atm}}$, since the water pressure acts on the balloon in the same way as on the piston. At the bottom, however, the pressure is increased due to the loaded piston mg/S, and secondly by the column of water above the balloon. So for V' we get

$$\begin{aligned} p_{\mathrm{atm}} V &= p'V' = (p_{\mathrm{atm}} + mg/S + h\rho g)V' \\ V' &= \frac{p_{\mathrm{atm}}}{p_{\mathrm{atm}} + mg/S + h\rho g} V \end{aligned} \tag{32.1}$$

This result has to be substituted into the relation for F' to get

$$F' = Mg - V'\rho g = (V - V')\rho g - F = \frac{V\rho g}{1 + \frac{p_{\mathrm{atm}}/g}{m/S + h\rho}} - F \approx 0{,}126\ \mathrm{N}. \tag{32.2}$$

33 The card at the bottom is our first card. If we want to lift it up a tiny bit, the total torque acting on that card must be zero with respect to the axis of rotation. That is in our case the edge of the card that is below all the other cards,

$$F\ell = \frac{1}{2}mg\ell + (\ell - \Delta\ell)F_1, \tag{33.1}$$

where F is the force that Justine exerts on the card and F_1 is the force that the second card (and all the other cards through it) exerts on the first card.

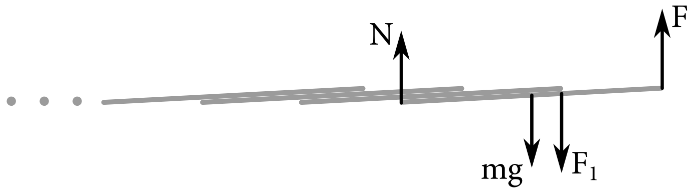


Figura 33.1: *Forces acting on cards*

For an infinite number of cards, we can assume that $F = F_1$. If, in fact, Justine removes the first card, there are still infinitely many cards she needs to lift up. With this assumption, we use the equation 33.1 to get

$$F = \frac{mg\ell}{2\,\Delta\ell}. \tag{33.2}$$

34 We shall consider the situation first in the non-inertial frame of reference with the observer in the axis of rotation in the roundabout's centre, rotating with the roundabout. In this frame of reference, the roundabout is motionless. There is a fictitious centrifugal force acting on Snow White given by $F_o = m\omega^2 r$ which she has to fight against to climb to the centre. Since Snow White does work, the total energy of the system is not conserved. However, since this force is purely radial, it has zero torque.

As Snow White starts moving, there is a second fictitious force at play: Coriolis force. This has the magnitude of $F_c = 2m\omega v_\perp$, where $v_\perp$ is the component of Snow White's velocity perpendicular to the axis of rotation. This force is perpendicular to both the axis of rotation and $v_\perp$, so it has nonzero torque. As Snow White climbs to the centre, she exerts a torque on the roundabout, transferring her angular momentum to the seven dwarves. As we've shown, the energy of the system isn't conserved, but the angular momentum is – this is clear when we look at it in an inertial frame of reference, since there is only one external force acting on the roundabout, and that is the force the ground exerts on the axle to hold it in place, which clearly has zero torque.

Since Snow White's angular momentum will be zero at the end of the manoeuvre, by equating the initial and final angular momentum we obtain

$$(m_s + 7m_t)\omega_1 r^2 = 7m_t\omega_2 r^2, \tag{34.1}$$

where m_s is Snow White's mass, m_t$ is the mass of one dwarf, ω_1 is the initial angular velocity and ω_2 is the final angular velocity.

We are given that $\omega_1 = \frac{2}{3}\omega_2$, which yields $(m_s + 7m_t) = \frac{3}{2}\cdot 7m_t$. This gives us the mass of one dwarf,

$$m_t = \frac{2}{7}m_s = 16\ \text{kg}. \tag{34.2}$$

35 Let us displace the disc from its equilibrium position by a distance Δx. The pressure in the compressed chamber of the cylinder before its base gets torn off will then increase by $\Delta p(\Delta x)$ and in the other chamber pressure drops by the same amount[6]. Therefore a force with magnitude $F_1(\Delta x) = 2\,\Delta p(\Delta x)S$ acts on the disc in an attempt to move the disc back into its equilibrium position (in the middle of the cylinder). In the small displacement approximation, the function $\Delta p(\Delta x)$ is linear.

In the cylinder with its base missing, the displacement of the disc by Δx from its equilibrium (which is still in the middle of the cylinder) causes the same difference in pressure $\Delta p(\Delta x)$ in the undisturbed chamber, but no difference in the other chamber. The force acting on the disc is now only

$$F_2(\Delta x) = \Delta p(\Delta x)S = \frac{1}{2}F_1(\Delta x). \tag{35.1}$$

This setup is in the first case equivalent to the disc being connected to a spring with appropriate stiffness k_1 and the other case is equivalent to the disc being connected to a spring with stiffness $k_2 = \frac{1}{2}k_1$. The period of small oscillations of a mass on a spring is $T = 2\pi\sqrt{\frac{m}{k}}$ with m being the mass of the disc and k being the spring stiffness, which means that the unknown ratio of the periods is

$$q = \frac{2\pi\sqrt{\frac{m}{k_1}}}{2\pi\sqrt{\frac{m}{k_2}}} = \sqrt{\frac{k_2}{k_1}} = \frac{\sqrt{2}}{2}. \tag{35.2}$$

[6] All of this is true only in the case of small displacements, because then the pressure as a function of displacement can be linearised.

36 How does the wheel accelerate after falling on the ground? Well, if the wheel has mass m, it's pressed onto the ground by gravitational force of size mg. Also, to the floor it seems as if the wheel is moving – since it's rotating with angular velocity ω_0, at the point of contact its surface moves at cross-radial velocity of $R\omega_0$ relative to both the wheel's centre of mass and the ground, where $R = 2{,}5$ m is the wheel's radius. This means there will be friction acting on the wheel at the point of contact with magnitude fmg (where f is the coefficient of friction) in the direction parallel to the ground and opposing the wheel's rotation. The wheel's centre of mass will begin linearly accelerating along the ground with its transverse velocity given by $v_x(t) = at = fgt$, but the friction also exerts torque that linearly decelerates the wheel's rotation, so that its angular velocity is given by

$$\omega(t) = \omega_0 - \frac{fmgR}{I}t, \tag{36.1}$$

where I is the moment of inertia of the wheel.

Friction, of course, doesn't act indefinitely – it will cease the moment the wheel stops slipping. This occurs once the cross-radial velocity at the surface $R\omega(t)$ equals the translational velocity $v_x(t)$, and the wheel reaches a stable rolling motion. Let's find the time t_v at which this occurs:

$$R\left(\omega_0 - \frac{fmgR}{I}t_v\right) = fgt_v \quad \Rightarrow \quad t_v = \frac{R\omega_0}{fg\left(1 + \frac{mR^2}{I}\right)}. \tag{36.2}$$

The final translational velocity of the wheel is then simply

$$v_v = v_x(t_v) = fgt_v = \frac{R\omega_0}{1 + \frac{mR^2}{I}}. \tag{36.3}$$

This is a known variable: what we want to find is the hamster's running speed before the breakage v_s, which is simply equal to the initial cross-radial velocity $R\omega_0$,

$$v_s = R\omega_0 = v_v\left(1 + \frac{mR^2}{I}\right). \tag{36.4}$$

We see that we still need to find the mass and the moment of inertia of the wheel. These can be found by linearly superimposing the properties of each metallic component. The moments of inertia of the 50 small rods on the circumference and circular hoops are trivial to calculate: they all lie at the same distance from the axis of rotation, and even if we cut them up into tiny parts, each part is at distance R from the axis of rotation. Therefore, their total moment of inertia is $50\lambda AR^2 + 2\lambda(2\pi R)R^2$, where A is the length of each small rod.

The two diametral rods are more tricky. Each has mass $2R\lambda$, and in literature we can find that the moment of inertia of a rod of length X and mass M rotating about an axis passing through its centre and perpendicular to the rod itself is $\frac{1}{12}MX^2$.

We have $X = 2R$ and by substituting the mass we find that each diametral rod has moment of inertia of $\frac{2}{3}\lambda R^3$. Hence the total moment of inertia of the wheel is

$$I = \lambda R^2\left(50A + \left(4\pi + \frac{4}{3}\right)R\right). \tag{36.5}$$

We swiftly express the wheel's mass from its dimensions and linear density as

$$m = \lambda(50A + (4\pi + 4)R), \tag{36.6}$$

hence the hamster's running speed is equal to

$$v_s = v_v\left(1 + \frac{50A + (4\pi + 4)R}{50A + \left(4\pi + \frac{4}{3}\right)R}\right) \doteq 2{,}11\ \mathrm{m/s}. \tag{36.7}$$

37 At first glance, it may seem that the satellite would leave the Sun's gravity field. However, if the force is sufficiently small, the satellite will still be bound to the Sun. Its trajectory will not be a conic section anymore but it will remain on some curve which will have a point with maximal radial distance from the Sun. We are looking for force F so that the maximum will be in radial distance of $2R$ from the Sun.

Before the engines started firing, the satellite was orbiting the Sun on a circular trajectory with radius R with speed of $v_0 = \sqrt{\frac{GM}{R}}$. After the engines started firing, the trajectory changed but the law of conservation of angular momentum still holds since the force F has only radial component. Let us denote v the speed of the satellite in distance $2R$. As this is the furthermost point from the Sun on the trajectory, the line joining the Sun and the satellite is perpendicular to its velocity. Therefore, we can write

$$v_0 R = v 2R \tag{37.1}$$

and thus

$$v = \frac{v_0}{2} = \frac{1}{2}\sqrt{\frac{GM}{R}}. \tag{37.2}$$

Next, we use the law of conservation of energy. We must be careful, since the force F does work, too. We divide the trajectory into many small pieces $\Delta\vec{s}$ and calculate the work done $\Delta W = \vec{F}\cdot\Delta\vec{s}$ by the force on the piece, and then sum all ΔW. If we decompose $\Delta\vec{s}$ into radial and transversal component, the dot product can be calculated as product of magnitude of the force $\vec{F}$ and magnitude of $\Delta\vec{s}$ as a result of properties of dot product. Therefore, the total work done by the engines from the start of their firing to the moment when the satellite reaches distance $2R$ from the Sun is equal to FR (the heliocentric distance of the satellite changes by R). Thusly, the law of conservation of energy can be written as

$$\frac{1}{2}mv_0^2 - G\frac{Mm}{R} = \frac{1}{2}mv^2 - G\frac{Mm}{2R} - FR. \tag{37.3}$$

Initial velocity is $v_0 = \sqrt{\frac{GM}{R}}$ and velocity in the distance $2R$ is expressed in 37.2. Now we simply solve for F. The result is

$$F = \frac{1}{8}\frac{GMm}{R^2}. \tag{37.4}$$

38 We need to realise what forces act on the golden sheet. Firstly, we have the gravitational force given by the Newton's gravitational law. Secondly, the reflected photons transfer momentum to the sheet. We use the alternate form of the Newton's second law $F = \frac{\Delta p}{\Delta t}$, where Δp is the difference in momentum. The difference in momentum of the photons that are reflected during a time Δt from the sheet is $\Delta p = \frac{2E}{c}$, where E is the energy incident on the sheet. The photons hit the sheet perpendicularly[7] and after reflection, they have an equal momentum in the opposite direction. Therefore the difference in momentum is doubled.

If the star radiates some energy E_c, the sheet, which is located in distance D, reflects

$$E = \frac{A}{4\pi D^2} E_c, \tag{38.1}$$

where A is the unknown area of the sheet.

The radiated energy can be determined from the Stefan-Boltzmann law for black body radiation as

$$E_c = P\,\Delta t = \sigma T^4 S\,\Delta t = \sigma T^4 4\pi R^2\,\Delta t. \tag{38.2}$$

The force exerted by the radiation must be equal to the gravitational force. This yields the equation

$$\frac{GMm}{D^2} = \frac{\Delta p}{\Delta t} = \frac{2E}{c\,\Delta t} = \frac{2A\sigma T^4 4\pi R^2}{4\pi D^2 c}, \tag{38.3}$$

from where

$$A = \frac{GMmc}{2\sigma T^4 R^2} \tag{38.4}$$

Noticeably, the area density does not depend on the distance from the star. This is an expected result, because both forces are proportional to D^{-2}.

39 This problem, as is usual in special relativity, can be approached in two ways: rigorously or with a trick. Let us denote the flight time of the journey's duration as $\tau = 2\,600\,000$ years and the distance between galaxy M31 and the Earth as $D = c\tau = 2\,600\,000$ light-years.

Rigorous approach

Let us consider the situation in two reference frames, writing down the spacetime coordinates of the Earth and galaxy M31 in each. In the Earth's frame of reference let the Earth be at $t = 0, x = 0$. Then, galaxy M31 is at $t = t_G, x = D$, where D is the given distance and $t_G = \frac{D}{v}$, where v is the traveller's velocity which we want to find. In the traveller's reference frame, these coordinates change under a Lorentz transform. The Earth is still at $t' = 0, x' = 0$, but galaxy M31 is now at

$$t' = \gamma\left(t - \frac{vD}{c^2}\right), \quad x' = \gamma(D - vt), \tag{39.1}$$

where $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$ is the Lorentz factor.

Since in this second reference frame the traveller is stationary, the difference of the time coordinates of the Earth and galaxy M31 (equal to t') is also the proper (and flight) time which the traveller measures as the duration of the journey – hence $t' = \tau$. Let us substitute the derived expression for t and express the traveller's

[7] we know this because $D \gg R$

velocity

$$\tau = \gamma D\left(\frac{1}{v} - \frac{v}{c^2}\right) = \frac{1}{v}\gamma D\left(1 - \frac{v^2}{c^2}\right) = \frac{1}{v}\gamma D\gamma^{-2} = \frac{D}{v\gamma},$$

$$\frac{D}{\tau} = \frac{v}{\sqrt{1-\frac{v^2}{c^2}}} = \sqrt{\frac{c^2}{\frac{c^2}{v^2}-1}}, \tag{39.2}$$

$$v = \frac{c}{\sqrt{1+\frac{c^2\tau^2}{D^2}}}.$$

But since $\frac{\tau}{D}$ is simply the inverse of the speed of light c^{-1} (this is how these quantities were given – we could have specified a different value of τ and the solution would be different), hence

$$v = \frac{c}{\sqrt{2}}. \tag{39.3}$$

Trick approach

Since we are interested in the flight time, which is the traveller's proper time, let us imagine we are aboard the travelling spaceship and consider which relativistic effects we observe. Clocks moving relative to us slow down, but that is not of interest to us. Moreover, the distances between points moving relative to us shorten by the Lorentz factor – and this is the effect that causes us to measure a shorter duration of our journey. So if we move at the velocity v, the observed distance to our destination is $D_0 = D\sqrt{1-\frac{v^2}{c^2}}$ and the journey duration is $\tau = \frac{D_0}{v}$. And since we are given $\tau = \frac{D}{c}$, we find

$$\frac{D\sqrt{1-\frac{v^2}{c^2}}}{v} = \frac{D}{c} \tag{39.4}$$

From this it is trivial to express the expected result we found in 39.3.

40 Let us consider a wall of thickness h, area S, and thermal conductivity λ. If we maintain its faces at temperatures t_1 and t_2, respectively, a heat flow

$$P = \frac{\lambda S}{h}(t_2 - t_1). \tag{40.1}$$

will be present.

To simplify matters, let us define coefficient $\lambda S/h =: X$. Now, let us look at a wall with two layers with coefficients X_1 a X_2, respectively. Continuity demands that

$$P = X_2(t_2 - t) = X_1(t - t_1), \tag{40.2}$$

where t is the temperature of the inter-layer interface. This temperature can be computed as

$$t = \frac{X_1 t_1 + X_2 t_2}{X_1 + X_2}, \tag{40.3}$$

and we can write an equation for the heat flow

$$P = \frac{X_1 X_2}{X_1 + X_2}\left(t_2 - t_1\right). \tag{40.4}$$

We can observe that a composite coefficient can be defined as

$$X_{12} \coloneqq \frac{X_1 X_2}{X_1 + X_2}. \tag{40.5}$$

With these preparations, we are ready to tackle the task itself.

Before styrofoam installation, the heat flow from Bob to neighbour was

$$P = \frac{X_{\text{floor}} X_{\text{ceiling}}}{X_{\text{floor}} + X_{\text{ceiling}}}\left(t_{\text{Bob}} - t_{\text{neighbour}}\right) = X_{\text{floor + ceiling}}\left(t_{\text{Bob}} - t_{\text{neighbour}}\right). \tag{40.6}$$

If we neglect neighbour's personal heat production, this heat flow is balanced with losses going outdoors

$$P = X\left(t_{\text{neighbour}} - t_{\text{outdoors}}\right), \tag{40.7}$$

where X is (for now) unknown coefficient describing heat losses to outdoors. With vital help of two equations above, we can determine it as

$$X = X_{\text{floor + ceiling}} \frac{t_{\text{Bob}} - t_{\text{neighbour}}}{t_{\text{neighbour}} - t_{\text{outdoors}}}. \tag{40.8}$$

After insulation, the coefficient describing heat flow between Bob's place and his neighbour becomes

$$X_{\text{floor + ceiling + styrofoam}} = \frac{X_{\text{floor + ceiling}} X_{\text{styrofoam}}}{X_{\text{floor + ceiling}} + X_{\text{styrofoam}}}. \tag{40.9}$$

Once again, the flow from Bob to neighbour is balanced with the losses, so

$$X_{\text{floor + ceiling + styrofoam}}\left(t_{\text{Bob}} - t'_{\text{neighbour}}\right) = X\left(t'_{\text{neighbour}} - t_{\text{outdoors}}\right) \tag{40.10}$$

Now, we can simply write

$$t'_{\text{neighbour}} = \frac{X t_{\text{outdoors}} + X_{\text{floor + ceiling + styrofoam}} t_{\text{Bob}}}{X + X_{\text{floor + ceiling + styrofoam}}}. \tag{40.11}$$

We can evaluate this result as 0,625 °C.

Respuestas

1 9

2 41 m

3 $\dfrac{5}{27}$

4 $0{,}073\ \mathrm{g/cm^3} = 73\ \mathrm{kg/m^3}$

5 $3{,}6\ \mathrm{s} = 10^{-3}\ \mathrm{h}$

6 $\dfrac{\sqrt{26}}{2}\ \mathrm{s} \doteq 2{,}55\ \mathrm{s}$

7 4,89 h = 17 591 s *or* 4 hours, 53 minutes, 11 seconds

8 134

9 4 : 3

10 7,68 l

11 151,6 m. *Accept results within the interval* 150 – 152 m.

12 2 Ω

13 1,2 m

14 98,3 kg, *accept results that fall within interval* 96 – 99 kg.

15 $\dfrac{\pi}{2}r^2 + r^2 = \left(\dfrac{\pi}{2} + 1\right)r^2$

16 203 l, accept also $0{,}2\ \mathrm{m^3} = 200\ \mathrm{l}$

17 $\dfrac{180^\circ}{2n+1},\ n \in \mathbb{N}_0$

18 $\sqrt{300}$ m/s ≐ 17,32 m/s. *Accept results in interval* 17,15 – 17,35 m/s.

19 $\frac{5\sqrt{3}}{4}$ s ≐ 2,17 s

20 314 °C

21 26,25 Ω

22 521 W

23 5618 kg/m^3, *accept results that fall within interval* 5612 – 5727 kg/m^3.

24 9 cm/s

25 *Accept results that fall within the interval* 1,08 – 1,10 m/s^2.

26 *Length* $L\frac{K}{k+K}$, *stiffness* $k+K$. *Accept only if both expressions are correct. If only one of the expressions is correct,* ***do not*** *indicate which one is wrong.*

27 $\sqrt{\frac{F}{ML}}$

28 170

29 2,2 MW

30 0,94 m

31 1,0 ns

32 0,126 N, *(first three significant digits)*

33 $\frac{mg\ell}{2\,\Delta\ell}$

34 16 kg

35 $\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

36 2,11 m/s

37 $\frac{GMm}{8R^2}$

38 $\frac{GMmc}{2R^2\sigma T^4}$

39 $\frac{c}{\sqrt{2}}$

40 0,625 °C