Dear readers,

in your hands you are holding the booklet of the 26th volume of Náboj Physics. The booklet contains all the physics problems you could have encountered during the competition this year, as well as the solutions to them, from which you can learn a lot. If you have any difficulty understanding any of them, do not hesitate to contact us; we will gladly clarify everything.

This booklet would not exist without the enormous effort of many people who participated in organising Náboj Physics. Most of us are students of Faculty of Mathematics, Physics and Informatics of Comenius University in Bratislava, and some of us also actively participate in organising the Physics Correspondence Seminar (FKS).

Náboj Physics continues with its international tradition. In the year 2023, Náboj Physics was held in Bratislava, Košice, Prague, Ostrava, Budapest, Gdańsk and Madrid. The results of this international clash can be found on our web site. For the international cooperation, we would like to thank to the local organisers: Patrik Rusnák (Bratislava), Marián Kireš (Košice), Jakub Kliment (Prague), Lenka Plachtová (Ostrava), Ágnes Kis-Tóth (Budapest), Brygida Mielewska and Kamil Żmudziński (Gdańsk) and José Francisco Romero García (Madrid).

In the name of the entire team of organisers, we believe that you enjoyed Náboj Physics in 2023, and we hope that we see each other at Náboj next year. Either as competitors or organisers.

Jaroslav Valovčan
Chief organiser

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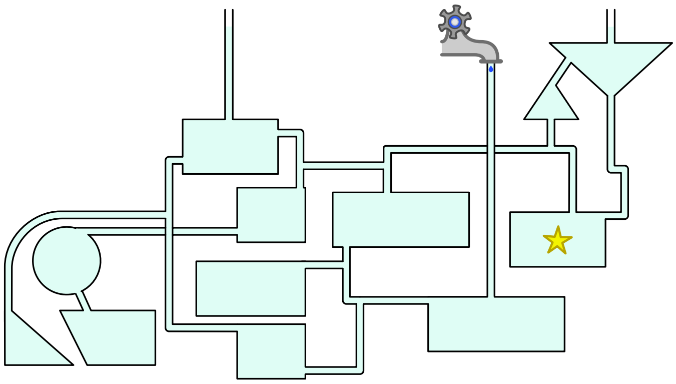
The results, the archive and other information can be found on the web site `https://physics.naboj.org/`.

Feladatok

1 Egy vízvezetéket újítanak föl nem messze Ádi házától. Ahogy Ádi elsétált a verejtékező munkások mellett, észrevette a vízvezetékek és tározók érdekes hálózatát, az alábbiak szerint. Ahogy a munkások elkezdik feltölteni a hálózatot, hány darab tározó lesz tele abban a pillanatban, amikor a megjelölt tározó csordultig megtelik?



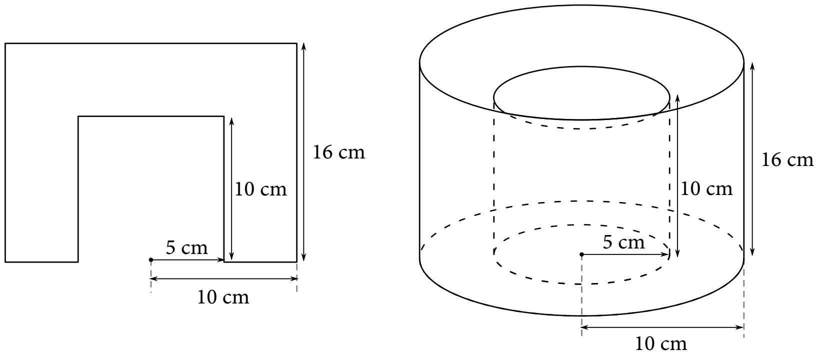
2 Átlagos Jánosnak 100 000 hajszála van, mindegyik szál évente 15 cm-t nő. Mennyit nő János haja összesen egy nap alatt?

3 Tamás a TV-t nézve egy Egyesült Államok-beli élő közvetítésre kapcsolt. A verseny kommentátora megjegyezte, hogy 200 mérföld per óra ugyanannyi, mint egy amerikai focipálya per másodperc. Tamást lenyűgözte az amerikai mértékegységrendszer komikussága, egészen addig, míg ő maga le nem ellenőrizte, hogy a kommentátor állítása messze esik a valóságtól. Mennyi focipálya per másodperccel tévesztette el a helyes értéket a kommentátor?

Egy amerikai focipálya 120 yard hosszú. Egy mérföld 1760 yardnak felel meg.

4 Még a *Ziggo & Marinelli*, a vezető savoyai piskótamorzsa gyártó vállalat sem kerülte el az infláció hatásait. A versenytársaikkal ellentétben nem növelték meg a termékeik árát, csupán áttervezték a csomagolás alakját. Az új csomagolás egy 10 cm sugarú, 16 cm magas üreges henger, míg az alapján egy 5 cm sugarú kerek részt megemeltek 10 cm-rel.

Mi a piskótamorzsa-keverék sűrűsége, ha a teli csomag tömege 350 g és egy üres csomagé pedig 40 g?

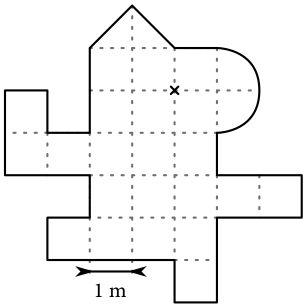


5 Gyuri oly nagyon szeret futni… hogy még egy okosórát is vett! Egy hónapnyi intenzív edzés után az óra megdícsérte a 17 km/h-ás futósebesség eléréséért. Sajnos most eleredt az eső, így Gyuri eldöntötte, hogy egy 2 m hosszú futószalagon folytatja edzését. Beállította a futószalag sebességét 5 m/s-ra és elkezdett futni pont a közepén. Mennyi idő múlva fog Gyuri leesni a futószalagról?

6 Pisztolyforgató Sebestyén a Vadnyugaton egy névtelen város főterén áll. A déli nap sugarában sütkérezne tovább, azonban észreveszi, hogy épp banditák próbálják meg bekeríteni. A pisztolyhős csak megfelelő figyelemmelterelésssel menekülhet meg a karmaik közül. Szerencsére a tegnap éjjeli dorbézolás után egy hatalmas pocsolya nagyon gyúlékony whisky maradt a főtéren.

Elsüti jó öreg revolverét a kereszttel jelölt pontra célozva. A whisky meggyullad, és a lángok 2 m/s-cel terjednek szét. Mennyi idő alatt kap lángra a teljes pocsolya?

A négyzetrács oldalai 1 m *hosszúak.*



7 A *Wings for Life* egy jótékonysági futóverseny, a következő szabályokkal: az összes versenyző egyszerre rajtol el. Fél óra elteltével egy autó is elindul utánuk, 14 km/h sebességgel és minden eltelt fél óra után automatikusan egy nagyobb sebességre kapcsol. Amikor egy versenyzőt megelőz az autó, számára a verseny véget ért.

A történelmi rekord táv, amelyet egy futónak sikerült megtennie így a versenyen, 92,14 km. Mennyi ideig futott a rekorder versenyző, ha az autó sebességértékei a verseny fél órás szakaszaiban rendre a következőképpen alakultak: 14, 15, 16, 17, 18, 22, 26, 30 és 34 km/h? A 34 km/h sebesség elérése után az autó nem gyorsult tovább.

8 Add meg az igaz állításokhoz tartozó számok összegét!

1	A lakásokban használt általános megszakítók névleges árama körülbelül 150 A.
2	A galvánelemek egyenáram források.
4	Lehetnek elektromos erőhatások szigetelők között.
8	Az áramvezetők ellenállása csökken melegítés hatására.
16	A coulomb mértékegység dimenzionálisan megegyezik azzal, hogy amper per másodperc.
32	1 A -nél kisebb áram nem veszélyes az emberre.
64	Két párhuzamosan kapcsolt ellenálláson lehet különböző feszültség.
128	Habár a konnektorból váltakozó áramot vehetünk, az elektromos vízforraló egyenárammal is tudna működni.

9 Éva legutóbbi Ádámnál tett látogatása után a lány elhatározta, hogy legközelebb kifesti a fiú szobáját, amíg ő nincs otthon. Erre a célra vásárolt egy vödör 5 literes festéket, amelyen havas sarkvidéki tájak képei és vakító fehérség ígéretére vonatkozó feliratok mellett a tartalmazott festék tömegét is feltüntették, méghozzá 7,5 kg értékben. A felhasználási javaslat pedig a következőképpen szól: 0,5 kg vizet keverjünk minden 1 kg festékhez.

Ám Éva túl lusta, hogy konyhai mérleggel szöszmötöljön, inkább egy merőkanalat szeretne használni a festék hígításához. Milyen **térfogati** arányban kell ekkor a vizet a festékhez kevernie?

A megoldást a festék térfogata:hozzáadott víz térfogata arányban adjátok meg!

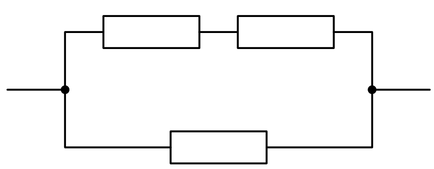
10 Ahogyan minden kommersz nyári sláger is hangsúlyozza, kiemelten fontos gondoskodni a megfelelő hidratáltságról és hűvösen tartani magunkat a forró hónapokban. Judit éppen a „Juci mami csókja“ nevű ízletes koktélt készíti, melyet 7 °C hőmérsékleten szoktak felszolgálni. Sajnos a csapból csak 20 °C hőmérsékletű víz folyik. Azért, hogy lehűtse azt, egy 1 kg tömegű jégkockát dobott az italba, amely 0 °C hőmérsékletű.

Mekkora a megfelelő hőmérsékletű víz térfogata, amelyet Judit így képes előállítani?

11 A tudomány nevében Máté és Sára eldötötték, hogy saját maguk fogják megtapasztalni a súlytalanságot egy hőlégballonból kiugorva. Amikor elértek 200 méter magasra, Máté izgatottan kivetette magát. Csak 3 másodperc múlva vette észre Sára, hogy Máté elfelejtette az ejtőernyőjét és felsikoltott ijedtében. Milyen magasan volt Máté, amikor meghallotta Sára sikoltását?

A hőlégballon nem mozdult. Tekintsünk el a közegellenállástól.

12 Megmértük három egyforma ellenállás ellenállását mind soros, mind párhuzamos kapcsolás esetén. Az eredmények 8 Ω-al tértek el. Mi az eredő ellenállása a képen látható hálózatnak?



13 Ferenc tűzoltó és épp egy magas épületről ereszkedik lefelé. Pont most ért el a 30 m hosszú kötele végére. Ha most minden erejét bevetve elrugaszkodna a faltól, 8,4 m vízszintes távolságra lengene ki attól.

Milyen magasra tud Ferenc a tűzoltó a földről függőlegesen felugrani?

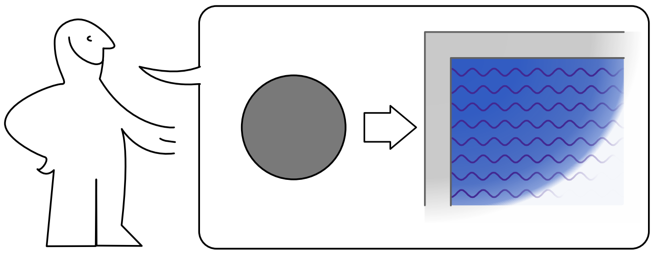
14 Jákó az autópályán száguld, amikor aggasztó rezgéseket érez az egyik kereke felől. Vegyünk egy egyszerű 10 g tömegű autógumi-szelepet. Mekkora látszólagos tömegűnek érezné az autógumi, ha 20 cm-re helyezkedik el a forgástengelytől, és az autó 500 km/h sebességgel halad? Tegyük fel, hogy a szelep távolsága a forgástengelytől sokkal nagyobb, mint a távolsága a kerék külső peremétől.

Egy tárgy látszólagos tömege, az a tömeg, amely a földön nyugalomban ugyanakkora erővel hatna egy mérlegre.

15 Júlia nyaralni ment. Az apartmanban, ahol megszállt, van egy téglalap alakú medence, aminek a méretei $a \times b$. Amikor becsekkolt, az apartman tulajdonosa megkérte, hogy esténként takarja le a medencét. Azonban Júlia csak egy kerek medencetakaró fedelet talált az apartmanban, aminek sugara $r \ll a, b$.

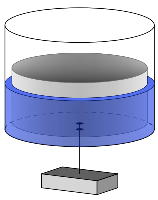
Júlia tisztában van vele, hogy nem fogja tudni letakarni a teljes medencét, de azért szeretné minél nagyobb felületét letakarni.

Mi a lehetséges legnagyobb felület, amit le tud takarni a medencéből úgy, hogy a fedél ne essen bele a medencébe?



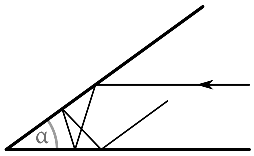
16 Marcellnak már van jogosítványa autóra és repülőre, és most szeretne egyet hajóra is. Azonban a ranglétra alján kell kezdenie, így egyelőre darukezelőként dolgozik a kikötőben. Minthogy elég élelmes, gyorsan kitalált egy új rendszert a konténerek pakolására. A rendszere egy könnyű 0,99 m sugarú henger, amely egy kicsit nagyobb, 1 m sugarú henger belsejében helyezkedik el. A kisebb henger aljának közepéhez egy rúd van rögzítve, amely áthalad a nagyobb henger alján, és a végén lévő kampóval tud konténereket felemelni. Marcell ezután a két henger közti résbe vizet tölt, míg a konténer el nem emelkedik a földtől. Mennyi vízre van szüksége egy 10 t tömegű konténer felemeléséhez?

Hagyjátok figyelmen kívül a víz viszkozitását és bárminemű szivárgást a rúd körül. A hengerek tömege elhanyagolható.



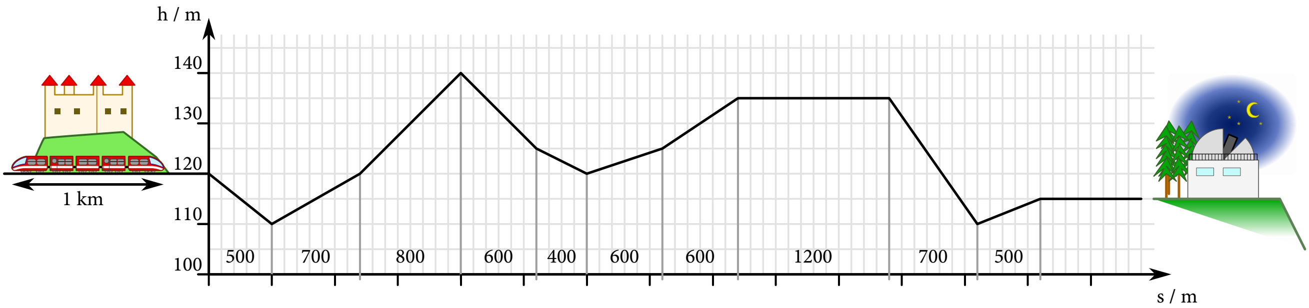
17 Irén egymás mellé helyezett két tükröt, úgy, hogy azok egy sarkot formázzanak α bezárt szöggel. Ezután megvilágította a sarkot egy lézer nyalábbal, az egyik tükörrel párhuzamos irányból. A sugár visszaverődött a tükrökről, majd kilépett a sarokból ezúttal a másik tükörrel párhuzamos irányban.

Milyen lehetséges értékekkel rendelkezhet a tükrök által bezárt szög?



18 Az elektromosság elég drága. Hogy pénzt spóroljon, a vasúttársaság arra utasította Jakabot a masinisztát, hogy ne gyorsuljon a dombon felfele menet és minél inkább használja ki a vonat lendületét. Jakab vonata 1000 m hosszú és 1000 t nehéz. Legalább mekkora sebességre kell felgyorsítania a vonatot a baloldali állomásból kiindulva, ha a képen látható út egészét a vonat lendületére hagyatkozva szeretné megtenni?

Jakab vonatának állandó a vonalmenti sűrűsége. A súrlódás és közegellenállás elhanyagolható. Nyugodtan használd a tan $x \approx x$ *közelítést kis szögek esetén.*



19 A Vadnyugaton Sebestyént, a pisztolyforgatót ismét banditák vették körbe a főtéren. Látványos menekülést tervez, mivel egy nagyon gyúlékony whisky pocsolya közepén áll, amely egy bunyóval és ivászattal eltöltött éjszaka maradványa. A pocsolya kerek, és a sugara 5 m.

Három töltényе maradt csak, hogy lángba borítsa az egész pocsolyát. Mi a minimum időtartam, ami szükséges ehhez? Minden revolverlövése meggyújtja a pocsolyát ott, ahol eltalálja, és a lángok 2 m/s-cel terjednek szét.

Sebestyént az egész Vadnyugaton villámkezűként ismerik, így az egy-egy lövése között eltelt idő, és a pisztolygolyók repülési ideje elhanyagolható.

20 Egy hideg téli napon, Dani két majdnem egyforma 1 m hosszú és 1 mm vastag fémszalagot talált a szemetes közelében. Csupán a két szalag hőtágulási együtthatója különbözött, az egyiké $8 \cdot 10^{-5}$ K^{-1}, míg a másiké 10^{-4} K^{-1} volt. A 0 °C fokos hidegben Dani egymásra hegesztette a két szalagot.

Milyen hőmérsékleteken fog a bimetál szalag gyűrűbe hajlani?

Tegyük fel, hogy a fémszalagok csak a leghosszabb tengelyükön tágulnak.

21 Andor valahonnan előkerített három ellenállást: kettő nyilvánvalóan azonos volt, a harmadik viszont eltérő. Mindhárom ellenállást felhasználva öt különböző elektromos áramkört épített, melyeknek eredő ellenállásaik 50, 60, 112, 140 és 245 Ω volt. Azonban nem vette észre, hogy még egy áramkört is lehetett volna készíteni.

Mekkora lenne ennek a hatodik áramkörnek az ellenállása?

22 A beeső napsugárzás teljesítménysűrűsége 1366 W/m^2. Ez mindazonáltal nem ragadtatja el Júliát, ugyanis utálja, amikor túlságosan felmelegszik a padlása. Ezért úgy döntött, hogy letakarja az ablakát egy különleges fényvisszaverő fóliával, ami csak a 80 %-át engedi át a beeső fénynek.

Az ablak egy 80 cm oldalhosszúságú négyzet. Júlia egy 1,5 m × 1 m-es, téglalap alakú fóliát vett, megfelelő alakú darabaokra vágta, és befedte az ablakot több rétegben, még ha így bizonyos helyeken több réteg is volt, mint máshol. Mi az a legkisebb teljesítmény, ami még így is átjuthat az ablakon, amikor a fény merőlegesen esik be az ablak síkjára?

23 Luca egy atléta, aki élete nagy magasugró megmérettetésére készül. Jelenleg annyi erővel tud ugrani, hogy testének tömegközéppontját pontosan 1 m-rel tudja megemelni. De ez Lucának nem elég. Ugrását egy kicsi, nem forgó, gömb alakú aszteroidán szeretné tökéletesíteni, melynek sugara 2,5 km. Vigyáznia kell azonban, nehogy akkorát ugorjon, hogy már ne essen vissza az aszteroida felszínére.

Mekkora az a maximális sűrűsége az aszteroidának, ami esetén Luca, ha függőlegesen felugrana, már soha nem térne vissza?

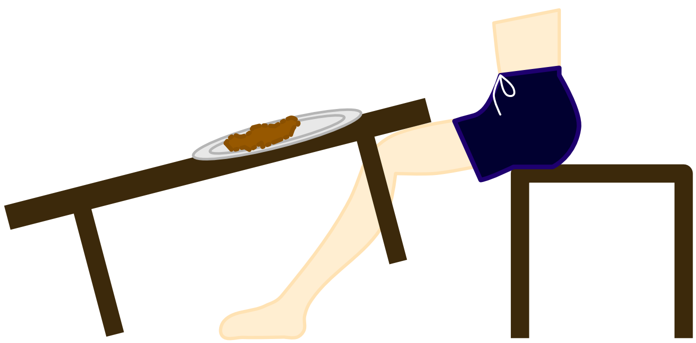
24 Cathy a földönkívüli sikeresen elnyert egy állami pályázatot, amellyel megépíthetett egy nagy távcsövet. A műszert jelenleg arra használja, hogy bolygókat keressen más csillagok körül. Például, ha a mi Naprendszerünk felé fordítja a távcsövet, láthatja, ahogy a Nap képe kicsit ingadozik ide-oda sugárirányban. Ez az 1 éves periódussal rendelkező, szinuszos mozgás bizonyíthatja a mi Földünk létezését, ahogy a Nappal együtt egy közös tömegközéppont körül kering.

Mennyi a Nap radiális sebességének amplitúdója, amely a Föld Nap körüli keringéséből származik?

Más bolygók hatásai elhanyagolhatóak. Cathy saját bolygója a Föld keringési síkjában található.

25 Ádám örül, hogy végre itt a vasárnap, és ebédre rántott húst ehet sültkrumplival. Ami viszont igazán kellemetlen a számára, az az, hogy a lábai olyan hosszúak, hogy amint leül az asztalhoz, az 30°-os szögben megdől. Így az asztalon lévő rántott húsos tányér elkezd csúszni, és a rántott hús is elkezd csúszni a tányéron. A tányér és az asztal közötti súrlódási együttható 0,4, és a tányér és a rántott hús közötti súrlódási együttható 0,3. A rántott hús tömege m, a tányér tömege pedig $2m$.

Mekkora a tányér gyorsulása közvetlenül azután, hogy Ádám leült?



26 Tominak van egy nulla nyugalmi hosszúságú, k merevségi tényezőjű rugója, és egy L nyugalmi hosszúságú, K merevségi tényezőjű, kicsit kisebb átmérőjű rugója. A kisebb rugót beletette a nagyobbikba, és a rugók két-két végét egymáshoz forrasztotta.

Mekkora lett a keletkező kettős rugó hossza és merevségi tényezője?

27 Sokszor, ha egy fizikai mennyiségnek csak a magnitúdóját szeretnénk gyorsan megbecsülni, érdemes a numerikus konstansokat elhanyagolni, és csak a mennyiség dimenzióját kiszámolni. Becsüljétek meg egy rezgő gitárhúr frekvenciáját ezen a módon, ha annak tömege M, hossza L, és a benne fellépő feszültség F.

28 Gyuri talált egy hisztogramot, amely megmutatja adott erősségű fölrengések évenkénti előfordulásainak számát. Az adathalmaz 1 szélességű intervallumokra volt osztva és Gyuri leolvasta róla, hogy 6200 darab 4,0 – 4,9-es erősségű és 800 darab 5,0 – 5,9-es erősségű földrengés történt. Viszont most azon töpreng, vajon hány darab 5,0-ös erősségű fordult elő.

Gyuri tudja, hogy a legalább M erősségű földrengések darabszámát a Gutenberg-Richter-törvény adja meg, vagyis

$$N(m \geq M) = 10^{a-bM},$$

ahol a és b konstansok.

Gyuri megfigyeléseire alapozva, számold ki, hány 5,0-ös erősségű földrengés fordult elő egy év alatt.

Egy földrengés erősségét 1 tizedesjegyre kerekítjük, ami azt jelenti, hogy az 5,0*-ös erősségű földrengések száma valójában az olyan erősségű földrengések pontos számára vonatkozik, amelyek a* $4{,}95 \leq m < 5{,}05$ *tartományba esnek. Hasonlóan, a* 4,0 – 4,9*-es erősségek tartománya a* $3{,}95 \leq m < 4{,}95$ *tartományra utal.*

29 Dani felfigyelt arra, hogy a kazánház másodpercenként 10 l, 80 °C-os rejtélyes folyadékkal biztosítja az épület fűtőrendszerét, azonban csak 9,9 l hűtött folyadék tér vissza a kazánhoz.

Mekkora a kazánház teljesítménye, ha a csővezeték sehol sem szivárog? A rejtélyes folyadéknak sűrűsége azon a ponton, ahol visszaérkezik a kazánházba 1000 kg/m^3. Térfogati hőtágulási együtthatója $1{,}8 \cdot 10^{-4}$ K^{-1} és fajhője 4000 J/(kg $\cdot$ K).

30 Amikor Patrik elköltözött, magával vitte a edzőgépét: egy nyugalmi helyzetben 1 m hosszú rugót. Ha a rugó egyik végét a mennyezethez rögzíti és a másik végébe belecsimpaszkodik, akkor a rugó pontosan 2 m-rel nyúlik meg.

Azon tűnődik Patrik, hogy milyen messze lenne a mennyezettől, ha a rugó mindkét végét a mennyezetre rögzítené, és pont a rugó közepéről csüngene le.

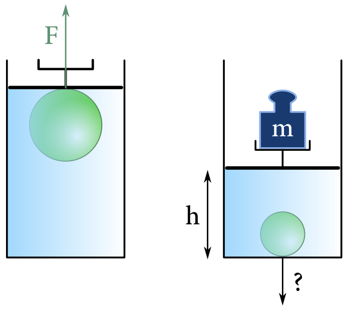
Legalább két értékes jegy pontosságú választ adj. Nyugodtan használj számológépet.

31 Bandi és Jákó elektronokat lőnek egymás felé. Bandi hirtelen egy kellemetlen helyzetben találta magát, konkrétan egy $v = 3200$ m/s-el haladó elektron egyenes útjában. Kénytelen a természetfeletti képességeihez folyamodni – rövid időre megidéz egy homogén mágneses teret, $B = 8{,}9$ mT mágneses indukcióval, a függőleges irányban (merőlegesen v-re).

Milyen sokáig kell fenntartania a mágneses teret, hogy eltérítse 90°-kal, és Jákót találja el helyette?

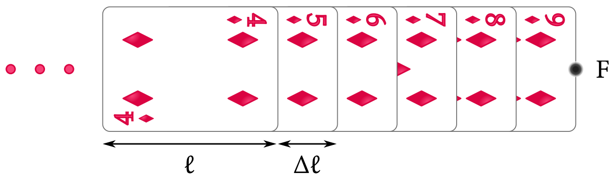
32 Egy függőleges henger alapfelülete $S = 50\ \text{cm}^2$, melyet vízzel töltöttek meg, és egy dugattyú zárja le felülről. Egy lufi, melynek térfogata $V = 50\ \text{cm}^3$, a hengerben található, a víz tetején lebeg, és 10 mN erőt gyakorol a dugattyúra felfelé.

Most egy $m = 10$ kg tömegű súlyt helyezünk a dugattyúra és a lufi lassan leereszkedik a henger aljára. Milyen erőt fejt ki alul, ha a dugattyú most $h = 1{,}8$ m magasságban van a henger aljához képest? Tegyük fel, hogy a kompresszió izoterm, és a lufi méretei nagyon kicsik a henger magasságához és szélességéhez képest. A hengert normál légköri nyomású levegő veszi körül.



33 Judit illúzionistának tanul éppen. Pókerarc gyakorlása, gondolatolvasás és kanálhajlítás mellett kártyavárakat is épít. Van egy megszámlálhatóan végtelen kártyapaklija, amiben a kártyák m tömegűek és a hosszuk ℓ. Ezeket úgy fekteti egymásra az asztalon, hogy mindegyik kártya el van tolva $\Delta\ell$-el az alatta lévőhöz képest.

Mekkora erőt kell kifejtenünk, hogy megemeljük a legalsó kártyát?



34 Amikor Hófehérke épp nem pihen, kedvenc elfoglaltsága a törpékkel egy tömegtelen körhintán lazulni. Ilyenkor 8 ülést foglalnak el, amelyek egyenlő távolságra helyezkednek el egymástól a körhinta kerületén és 3 másodpercenkénti 1 fordulatszámmal pörgetik meg azt.

Egyszer csak egy alma pottyan a körhinta kellős közepére. Hófehérke felkel az üléséből, hogy felszedje. Amint beér az almáért középre, a körhinta már 2 másodpercenként 1 fordulatszámra vált.

Mekkora 1 törpe tömege, ha Hófehérke 56 kg?

Tegyük fel, hogy a hét törpe mindegyike ugyanannyit nyom.

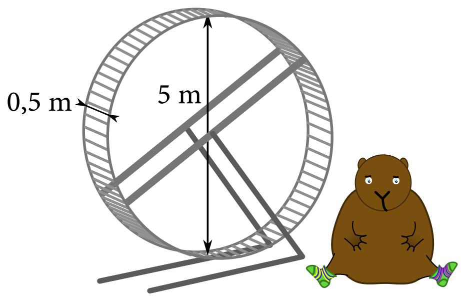
35 Szabinának van egy üreges, 10 ml térfogatú hengere, melynek alapterülete 1 cm^2. A henger levegővel van megtöltve, légköri nyomáson. Egy szabadon mozgó 100 g tömegű vékony korong légmentesen két részre osztja a hengert.

Szabina nem rendelkezik megfelelő képesítéssel egy ilyen kényes műszer kezelésére, és egy kis idő múlva valahogy sikerül lefeszítenie az egyik talpat. Mekkora volt a korong rezgésidejének aránya az alap lefeszítése előtt és után?

Vegyük a levegőt egy ideális kétatomos gáznak. Szabina normál légnyomáson tartózkodik.

36 Egy túl nagyra nőtt hörcsög vadul sprintel a mókuskerekében. Hirtelen meghaladja a zsanérok teherbírási kapacitását és leesik a kerék. A hörcsög szerencsére kiesik a kerékből, mielőtt az a földre esve felgyorsul 1 m/s stabil gurulási sebességre. Milyen gyorsan futott a hörcsög, ha kezdetben nyugalomban volt a szobához képest?

A mókuskerék λ vonalmenti sűrűségű fémrudakból van építve. A kerületét egy pár 5 m átmérőjű fémkarika adja ki, amelyek ötven egyenlően elhelyezett 0,5 m hosszú rúddal vannak egymáshoz erősítve. A karikák egy-egy, átmérő hosszúságú rúddal vannak a tengelyhez csatlakoztatva, merőlegesen keresztezve azt.



37 Egy m tömegű műhold kering az M tömegű Nap körül, R sugarú körpályán. Egy ponton beindítja a motorját, amely állandó F erővel tolni kezdi, amely mindig sugárirányban a Naptól elfelé mutat.

Mekkora erőre van szükség ahhoz, hogy a műhold maximum $2R$ távolságra érjen el a Naptól?

38 Az arany egy kifejezetten képlékeny anyag és nagyon vékony lemezeket lehet verni belőle. Továbbá kémiailag stabil és a fényt is nagyon jól veri vissza. Patrik az Űrhódító épp megérkezett egy újonnan felfedezett csillaghoz, ami T hőmérséklettel, M tömeggel és R sugárral rendelkezik. A jövő civilizációinak üdvözlésére ott szeretne hagyni egy levelet, amit tiszta, m tömegű és ρ sűrűségű aranylemezbe vésett.

Mekkora felületi sűrűségű lemezt kell az Űrhódítónak vernie ahhoz, hogy a sugárzás egy konstans $D \gg R$ távolságra tartsa a lemezt a csillagtól?

Feltételezzük, hogy a lemez a fény egészét visszaveri és a beérkező sugárzás merőlegesen érkezik a lemezre.

39 Az Androméda galaxis 2 600 000 fényév távolságra van tőlünk. Milyen gyorsan kéne odautaznunk ahhoz, hogy az út megtétele a mi szemszögünkből nézve ugyanennyi évig tartson?

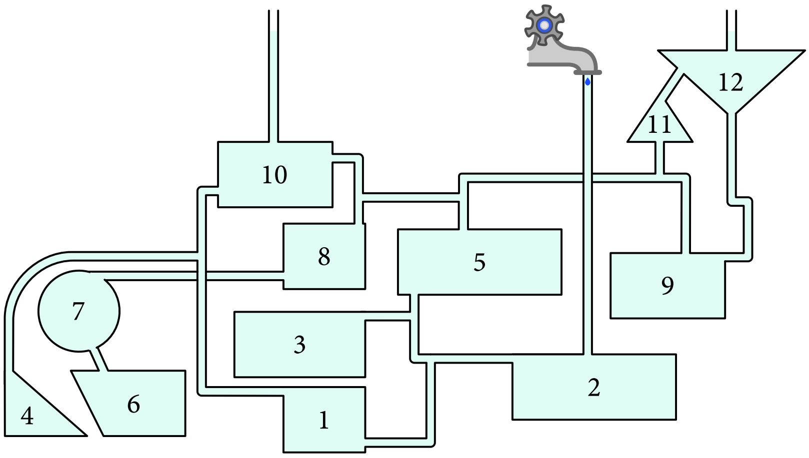
40 Sztereotípiák ellenére, sok kedves emberrel találkozhatunk nagyvárosokban… Ugyanakkor ott van Robi. Robi nem szereti a felette élő szomszédját, mivel az még -5 °C-os téli hidegben sem kapcsolja be a fűtést. Robi mennyezete 20 cm vastag beton, 20 W/(m · K) hővezetési tényezővel. A mennyezetre egy 1 cm vastag fapadlózat van fektetve, 2 W/(m · K) hővezetési tényezővel.

Robi 25 °C-on szereti tartani az lakását, viszont ezzel 10 °C-ra fűti fel szomszédjáét. Mekkora lesz a szomszédja lakásán a hőmérséklet miután Robi egy 5 cm vastag hungarocell réteget erősít a mennyezetéhez? A hungarocell hővezetési tényezője $1\ \mathrm{W}/(\mathrm{m}\cdot\mathrm{K})$, a plafon $10\ \mathrm{m}^2$ területű.

Robi állandó hőmérsékleten tartja lakását. A padlótól eltekintve a felső szomszéd lakásának csak külső falai vannak.

Megoldások

1 The picture below shows the order in which the tanks are filled. Due to gravity, the water flows down and fills the lowest tank numbered 1. Then the water moves up and displaces the air. The water level in the tanks as well as in the pipes rises, and we fill tanks 2 and 3. Then the triangular tank number 4 fills up, followed by the tank 5. As the water level rises further, the water fills the branch with tanks 6, 7 and 8, and finally the starred tank is filled as ninth.



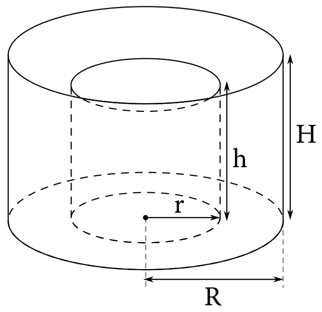
1.1. Ábra: *Tank filling order*

2 If one hair grows 15 cm/y, it will grow $\frac{15}{365}$ cm in one day. To get the answer, we simply multiply this by the total number of hairs and convert to metres to obtain $100000 \cdot \frac{15}{365}$ cm/d $\doteq 41$ m/d.

3 If we travel 200 miles per hour, which is $200 \cdot 1760 = 352\,000$ yards per hour, in one second we move $\frac{352000}{3600} = \frac{880}{9}$ yards. The commentator said that this distance is one football field and he is wrong by $120 - \frac{880}{9} = \frac{200}{9}$ yards. In units of football fields this is simply $\frac{200}{9 \cdot 120} = \frac{5}{27}$ of a football field per second.

4 The density is $\rho = \frac{m}{V}$. If the container weighs m, the mass of the biscuits m_b (without the container m_0) is $m_b = m - m_0$. The volume of the container is the difference between the volumes of the cylinder and the raised circular part of its base,

$$V = H\pi R^2 - h\pi r^2. \tag{4.1}$$



4.1. Ábra: *The container with dimensions shown*

The density of the biscuits is therefore

$$\rho = \frac{m_b}{V} = \frac{m - m_0}{H\pi R^2 - h\pi r^2} \doteq 73\ \mathrm{kg/m^3}. \tag{4.2}$$

5 Firstly, let us convert George's velocity to metres per second:

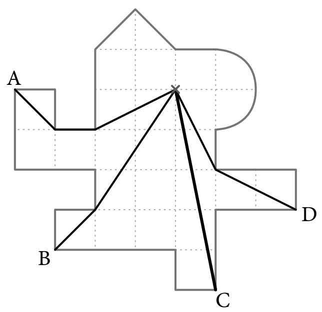
$$\frac{17\ \mathrm{km}}{1\ \mathrm{h}} = \frac{17\,000\ \mathrm{m}}{3600\ \mathrm{s}} = \frac{170}{36}\ \mathrm{m/s}. \tag{5.1}$$

When he steps on the treadmill going at the speed $\frac{180}{36}$ m/s, the difference of the speeds is $\frac{10}{36}$ m/s.

The time needed for him to fall off the treadmill is the time it takes him to cover 1 m at the speed of $\frac{10}{36}$ m/s, so

$$\frac{1\ \mathrm{m}}{\frac{10}{36}\ \mathrm{m/s}} = 3{,}6\ \mathrm{s}. \tag{5.2}$$

6 The whole puddle will be engulfed in flames, when fire reaches the furthermost point. This point lies on the circumference of the puddle and we immediately see some candidates.



6.1. Ábra: *Candidates for the furthermost point*

Using the Pythagorean theorem, we calculate that the furthermost point is C located $\sqrt{26}$ m from the point of ingition. Fire spreads at speed of 2 m/s, therefore whole puddle will be in flames $\frac{\sqrt{26}\ \mathrm{m}}{2\ \mathrm{m/s}} \doteq 2{,}55$ s after ignition.

7 Tegyük fel, hogy a futó és az autó egyszerre indulnak el, csak az első fél órában az autó sebessége 0 km/h. Ekkor meg tudjuk határozni a 92,14 km megtételéhez szükséges időtartamot. Az első 9 fél óra alatt az autó összesen

$$0\ \mathrm{km} + 7\ \mathrm{km} + 7{,}5\ \mathrm{km} + 8\ \mathrm{km} + 8{,}5\ \mathrm{km} + 9\ \mathrm{km} + 11\ \mathrm{km} + 13\ \mathrm{km} + 15\ \mathrm{km} = 79\ \mathrm{km}. \tag{7.1}$$

távolságot tesz meg.

Ezután az autó már egyenletes 34 km/h-s sebességgel halad, és ahhoz, hogy a futót utolérje, még 13,14 km-t kell megtennie. A teljes ehhez szükséges idő tehát

$$9 \cdot 0{,}5\ \mathrm{h} + \frac{13{,}14\ \mathrm{km}}{34\ \mathrm{km/h}} \doteq 4{,}89\ \mathrm{h}. \tag{7.2}$$

8 Since the numbers are all different powers of two, you have to determine the truthfulness of all statements.

***Common circuit breakers in an apartment have a rated current nominal of about* 150 A.**
The statement is **false**. Standard values are around 16 A. For the voltage in a mains socket 230 V it means a power limit of 3500 W, so four hoovers vacuuming simultaneously would trip the breaker. Breakers at 150 A would easily handle 20 hoovers, which disagrees the common experience.

Galvanic cells are sources of direct current.
The statement is **true**. Galvanic cells, such as a common AA battery, produce electric energy by electrolytic decomposition, which is a process with a given cathode and anode. Therefore, direct current is produced.

Electric forces can occur between insulators.
The statement is **true**. A good example is an electroscope, a basic demonstration of electrostatic force, where two charged plastic balls repel. Therefore, even insulators can be charged on the surface. Have you ever rubbed a balloon against your hair?

Resistivity of electric conductors decreases when they heat up.
The statement is **false**. You can easily search up values of resistivity at different temperatures for any type of conductor in literature. For the majority of common materials, their resistivity increases with rising temperature.

The coulomb is dimensionally equivalent to ampere per second.
The statement is **false**. Electric current, measured in amperes, tells us how much charge (coulombs) passes through per unit time, therefore it's the other way around – amperes are coulombs per second.

Currents smaller than* 1 A *are not dangerous to humans.
The statement is **false**. Even currents as small as 20 mA can be dangerous to humans, especially if passing through one's heart and internal organs. This isn't necessarily related to the current passing through the circuit before we touch it, though. The resistivity in the circuit can be small, therefore huge current can be passing through it even at small voltage. If the resistivity of a human body is large, a small amount of current would pass through it from that small voltage. Conversely, if the voltage is great, but the source cannot

produce a large current, we're once again in the clear. A typical example is spark discharge, which can even have 10 000 V, but the transferred charge, and therefore also current, are small.[1]

Two resistors connected in parallel can experience different voltages.

The statement is **false**. Two resistors in parallel are have their ends connected. We know that two points connected by a wire have zero electric potential between them, so the ends of the two resistors have the same voltages. The voltages across both resistors will be the same. A real-life example would be a lightbulb. We often use multiple lightbulbs, with the intention of having each at a 230 V voltage – and this is why they're always in parallel. Even electric sockets are connected in parallel; if we connected two lightbulbs in series, each of them would have half the voltage from the socket, and they would shine at a lower power.

Even though mains sockets provide alternating current, electric kettles would work with direct current as well.

The statement is **true**. An electric kettle is just a jug with a resistive spiral connected to a bimetallic thermal switch… and in a simpler model, it is just a switch and a resistor that heats up and transfers the heat to the water. Resistors heat up under both direct and alternating currents, so a kettle would work just the same even when connected to a source of direct current.

The sum of the numbers of all true statements is therefore 2 + 4 + 128 = 134.

$\boxed{9}$ Ha 5 l festék 7,5 kg, akkor a sűrűsége $\frac{7{,}5\ \mathrm{kg}}{5\ \mathrm{l}} = 1{,}5\ \mathrm{kg/l}$. Tudjuk, hogy a víz sűrűsége 1 kg/l, vagyis a sűrűségek aránya 3 : 2. Vagyis 1 tömegegységnyi festék és 1 tömegegységnyi víz térfogatának aránya ennek fordítottja: 2 : 3 lesz. Ha még egy tömegegységnyi festéket hozzáteszünk, vagyis 2 : 1 arányban vizsgáljuk a két összetevőt, amelyet a festékes dobozon található instrukció is ajánl, 4 : 3 térfogati arányra jutunk, amely a végső megoldás.

$\boxed{10}$ Granny Justine has to pour mass m of tap water in a jug in which she then puts a block of ice of mass $m_{❄}$. In order to melt the whole icy block, it has to absorb heat equal to $m_{❄} l_{❄}$, where $l_{❄}$ is specific latent heat of fusion of water. Then, the melted ice, also known as water, heats by $\Delta T_{❄}$, so it absorbs heat $m_{❄} c_{\mathrm{H_2O}}\, \Delta T_{❄}$, where $c_{\mathrm{H_2O}}$ is specific heat capacity of water. All this heat is absorbed from the tap water, which cools by $\Delta T_{\mathrm{H_2O}}$ so it gives away heat of $m c_{\mathrm{H_2O}}\, \Delta T_{\mathrm{H_2O}}$ to the ice.

Heat absorbed by the ice is equal to heat given away by the tap water, thus

$$m_{❄} l_{❄} + m_{❄} c_{\mathrm{H_2O}}\, \Delta T_{❄} = m c_{\mathrm{H_2O}}\, \Delta T_{\mathrm{H_2O}}. \tag{10.1}$$

We solve for mass of the tap water

$$m = \frac{m_{❄} l_{❄} + m_{❄} c_{\mathrm{H_2O}}\, \Delta T_{❄}}{c_{\mathrm{H_2O}}\, \Delta T_{\mathrm{H_2O}}}. \tag{10.2}$$

The values from the problem statement give $m \doteq 6{,}68$ kg. Apart from tap water, the jug contains also 1 kg of water which stems from the ice, so, all in all, there is 7,68 kg of water in the jug which is 7,68 l.

[1] A common question goes: who's the killer, amperes or volts? Do you know?

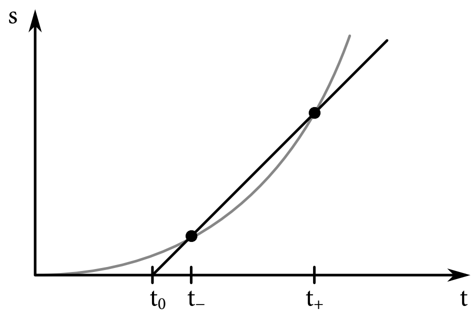
11 Let t_0 denote the time of Sarah's scream. The main idea here is that Sarah's sound, which propagates at a constant velocity c, will overtake linearly accelerating Matt at some time t at which the distances traversed by both Matt and the sound of the scream will be the same

$$\frac{1}{2}gt^2 = c(t - t_0)$$
$$\frac{1}{2}gt^2 - ct + ct_0 = 0. \tag{11.1}$$

This is a quadratic equation with two roots

$$t_\pm = \frac{c \pm \sqrt{c^2 - 2gct_0}}{g}. \tag{11.2}$$

The numerical values of these roots are $t_+ \approx 66{,}79$ s and $t_- \approx 3{,}14$ s respectively. Physically, these correspond to when the sound first overtakes Matt and, much later, when constantly accelerating Matt overtakes the sound again (since we're disregarding air friction). Hence $t = t_-$.



11.1. Ábra: *Distances travelled by Matt and Sarah's scream*

Now that we know time t, we can substitute it into the equation of linearly accelerated motion

$$s = \frac{1}{2}gt_-^2 \approx 48{,}4 \text{ m}. \tag{11.3}$$

Matt jumped from the height $h = 200$ m, therefore at time t when he hears Sarah's roar he will be at height of $h - s \approx 151{,}6$ m.

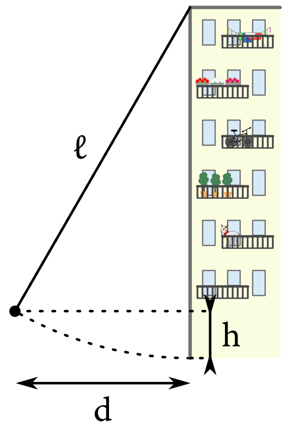
12 Let us denote the value of the resistor's resistance R. If we have a series connection, we sum the resistances of the resistors, so the total resistance will be $3R$. In parallel connection, we are summing reciprocal values of the resistances, so the resistance is equal to $R/3$. These values differ by 8 Ω, so we can construct the equation

$$8\ \Omega = 3R - \frac{R}{3}. \tag{12.1}$$

The solution of this equation is $R = 3\ \Omega$. The diagram in the figure in the problem statement is a parallel connection of two branches with resistances R and $2R$, so its total resistance can be calculated as

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R} \quad \Rightarrow \quad R' = 2\ \Omega. \tag{12.2}$$

13 Francis uses the same amount of kinetic energy for his jump and push off the wall. In the highest point of his jumps, all kinetic energy will change to potential energy which will be the same in both cases. It means that the height of the jump will be equal. The height can be determined from the picture geometry as $h = \ell - \sqrt{\ell^2 - d^2} = 1{,}2$ m.



13.1. Ábra: *Geometry of Francis' jump*

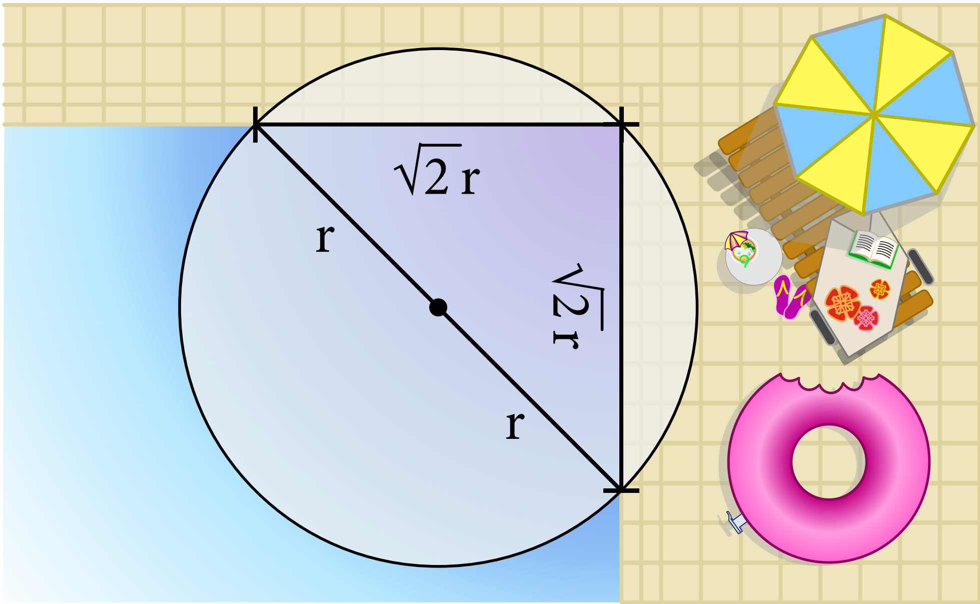
14 If the wheel rotates, the tyre valve rotates with it. It means that in the frame reference of the rotating wheel with the origin in the centre of the tyre the valve stands still. The fictitious centrifugal force acting on the valve is $F = \frac{mv^2}{r}$ which is oriented from the origin. However, the valve is not moving, so there must be opposite force with the same magnitude from the wheel. When the car accelerates, in the frame reference of the wheel it seems that the valve changes its weight, however the only thing that changes is the frequency of rotating wheel. The apparent weight can be computed as

$$m_a = \frac{F}{g} = \frac{mv^2}{rg}.$$

After the unit conversion from km/h to m/s and using the given values we get $m_a \approx 98{,}3$ kg.

15 It is necessary to realize that if Julia has only one cover, she will cover the greatest area of the pool if she places it into the corner of the pool so that its centre protrudes over the water as much as possible. This situation is depicted in the figure below. The centre of mass must be located on the line segment connecting the two points that are the intersections of the cover's circumference with the perimeter of the pool. In this case, the corner of the pool also lies on the circumference of the cover.[2] If Julia moved the cover a little more over the water, it would fall into the pool. Therefore it is enough to calculate the area of water that is covered. This area consists of a semicircle and a right triangle. The area of a semicircle is simply $\frac{1}{2}\pi r^2$.

[2] thanks to Thales's theorem



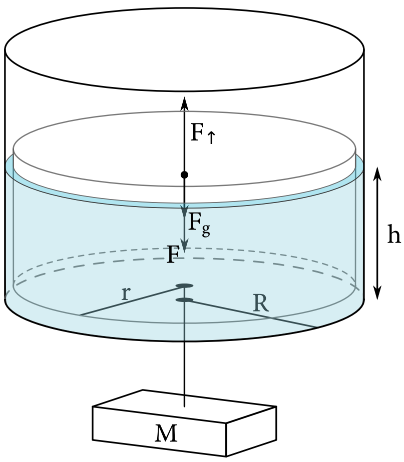
15.1. Ábra: *The cover in optimal location*

We know that the right triangle's hypotenuse has a length of $2r$ and its other two sides have a length of $\sqrt{2}r$ because the Pythagoras' theorem must hold: $\sqrt{\left(\sqrt{2}r\right)^2 + \left(\sqrt{2}r\right)^2} = 2r$. The area of this triangle is $\frac{1}{2}(\sqrt{2}r)^2 = r^2$. The total area covered by the circle is then

$$\frac{1}{2}\pi r^2 + r^2. \tag{15.1}$$

16 Let us draw and write down all the forces that act on the axis of the cylinder:

$$F_g + F = F_{\mathrm{vz}}, \tag{16.1}$$



16.1. Ábra: *Marcel's crane with forces* where F_g is the gravitational force, F is the force from the container, and F_{vz} is the buoyant force of the water. After expressing the individual forces, we get

$$mg + Mg = \rho_{H_2O} g h S_1, \tag{16.2}$$

where h is the height of the submerged part of the cylinder and S_1 is the base of the smaller cylinder. To find the volume, we need to find h,

$$h = \frac{m + M}{\rho_{H_2O} S_1}. \tag{16.3}$$

We know that the mass of the smaller cylinder is negligible compared to the container, so

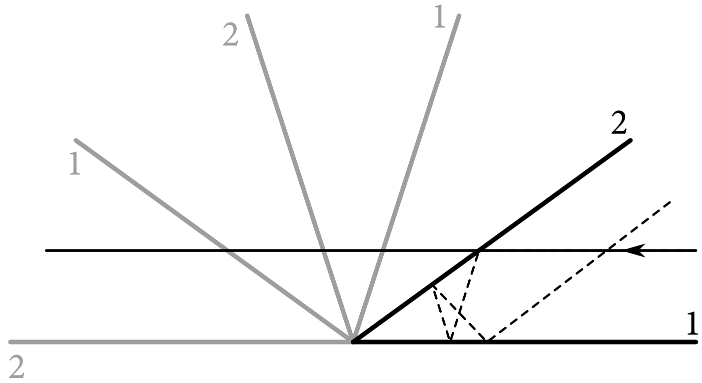
$$h \approx \frac{M}{\rho_{H_2O} S_1}. \tag{16.4}$$

The volume of water will eventually be just the volume difference of the cylinders, so

$$V = h(S_2 - S_1) = \frac{M}{\rho_{H_2O} r^2}(R^2 - r^2) \doteq 203\,\mathrm{l}. \tag{16.5}$$

$\boxed{\mathbf{17}}$ Instead of calculating complicated geometry, we realise that we just need to mirror the space between the mirrors a few times. Then we can consider that a ray of light travels along a straight trajectory as in the picture.

At the end, the ray of light must be parallel to the other mirror. This means that the angle 180° must be divided into a whole number of parts by the angles α. Thus, it would seem that the valid values of α would be $\frac{180°}{n}$, where n are natural numbers. However, for the ray at the end to be parallel to the other mirror it is necessary, that the mirror which appears at an angle 180° is the other mirror.



17.1. Ábra: *Solution with just four reflections*

We quickly realise that this condition tells us that the number n must be odd. Therefore the valid values of the angle α are

$$\alpha = \frac{180°}{2n + 1}, \tag{17.1}$$

where $n \in \mathbb{N}$.

18 The train starts off with speed v and thus has a certain amount of kinetic energy. Since neither friction, nor air resistance play a role, total mechanical energy of the train is conserved. Therefore it only needs to travel at such a speed so that its centre of mass just passes over the highest point of its trajectory. However, since the train is not a point mass, the trajectory of its centre of mass differs from the altitude profile of the track. At a glance, we can determine that two areas come into consideration as highest: the peak at 140 m and the plateau at 135 m. Obviously, there is no way or place to get any higher.

When we calculate the slopes, we can see that they are all quite small, the steepest drop is only $\frac{25}{700}$. For small angles we can neglect the difference between the actual length of the train and its projection onto the horizontal plane, which will simplify the calculation somewhat. Then we can also ignore the height of the train's centre of gravity above the rails, as this will always be the same.

Let us first look at the highest point of the track. The slopes on both sides are equal, 25 ‰, so the train's centre of gravity will be at the highest point when it passes over the summit. Since the linear density of the train is constant, the centres of gravity of both the front and rear halves of the train are at equal altitudes, and hence the centre of gravity of the whole train is at that altitude as well. So we just need to calculate the height of the centre of gravity of one of the halves, which will be

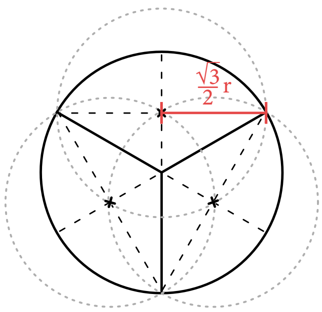
$$h_\wedge \approx \frac{140\text{ m} + (140\text{ m} - 0{,}025 \cdot 500\text{ m})}{2} = 133{,}75\text{ m}. \tag{18.1}$$

However, this is obviously less than the height of the straight part on the right, where the train fits entirely. The height of its centre of gravity on it will therefore be trivially $h_- = 135$ m.

All that remains is to express the difference in the heights of the centre of gravity at the start and at the maximum height, which is

$$\frac{1}{2}mv^2 = mgh_- \quad \Rightarrow \quad v = \sqrt{2gh_-} \approx \sqrt{2 \cdot 10\text{ m/s}^2 \cdot 15\text{ m}} = \sqrt{300}\text{ m/s} \doteq 17{,}3\text{ m/s}. \tag{18.2}$$

19 Let us denote v the speed at which fire spreads, and t the time when the whole puddle is burning. Our task is to cover the puddle with three circles with radius vt and optimalise their placement to minimise t. We divide the puddle into three sections, each being inflamed by one of the pistolero's bullets. We want these sections to be as small as possible, which means we want to make the circle covering it as small as possible. Symmetry of the puddle allows us to divide it into three equally sized circular sectors and a simple thought convinces us that it is not possible to find a better solution. Ideal points of bullet impacts can be found as centres of the circles.



19.1. Ábra: *Anatomy of Sebastian's inflammatory activity*

A short exercise from geometry reveals us that the longest distance which has to be travelled by flame has length of $\frac{\sqrt{3}}{2}r$, where r is radius of the puddle. Entire puddle will therefore be engulfed in flames in time $\frac{\sqrt{3}}{2}\frac{r}{v} = \frac{5\sqrt{3}}{4}\ \mathrm{s} \doteq 2{,}17\ \mathrm{s}$.

20 Since the thickness of the strips is one thousand times smaller than their length, we can neglect the changes in their thickness due to temperature changes and assume that only their length varies. Let r_1 and r_2 be the radius of the inner and outer strip, respectively. If we denote their thickness h, it then follows $r_2 = r_1 + h$. The strip with original length l_0 and thermal expansion coefficient α will, due to a temperature difference ΔT, increase its length by $l_0\alpha\,\Delta T$. Its new length will therefore be

$$l_0 + l_0\alpha\,\Delta T = l_0(1 + \alpha\,\Delta T). \tag{20.1}$$

If the strip has a circular shape with radius r, its length is equal to $2\pi r$. For Daniel's newly acquired strips this can be expressed as

$$\begin{aligned} l_0(1 + \alpha_1\,\Delta T) &= 2\pi r_1, \\ l_0(1 + \alpha_2\,\Delta T) &= 2\pi r_1 + 2\pi h, \end{aligned} \tag{20.2}$$

using $r_2 = r_1 + h$. We can subtract the second equation from the first and we get

$$\Delta T = \frac{2\pi h}{l_0(\alpha_2 - \alpha_1)}. \tag{20.3}$$

This means we have to either cool down or warm up the two strips by ΔT. For given values $\Delta T \doteq 314\ \mathrm{K}$. Since the dimensions of the strips were measured at $0\ °\mathrm{C} \doteq 273\ \mathrm{K}$, we have to increase the temperature by ΔT, since it is impossible to achieve negative thermodynamic temperature. The bimetallic strip will take circular shape at the temperature 314 °C.

21 When solving this problem we will not avoid guessing. This does not mean that we can't guess wisely. In total, there are six possible circuits. We will guess that the purely parallel and the purely serial wiring has the smallest and the largest resistance, respectively. These have following resistances

$$\frac{R_1 R_2}{R_1 + 2R_2} \qquad \text{and} \qquad 2R_1 + R_2, \tag{21.1}$$

where R_1 is the resistance of two identical resistors and R_2 is the resistance of the third resistor.

But if we consider that the smallest and the largest resistance is 50 Ω and 245 Ω, we get a system of two equations that has no solution. This means that the value we are missing must be just the smallest or the largest. So we need to guess which circuit corresponds to the second smallest, or the second largest resistance, respectively.

Firstly, we can notice that the resistors in parallel always have lower resistance, than the individual resistors, and when connected in series higher resistance. In the case of two parallel branches, the resulting resistance is always lower than the resistances in the individual branches. In the case of a series connection, the resulting resistance is higher than the individual resistors connected in series. This allows us to assume that the smaller resistances correspond to parallel wiring and larger resistances to serial wiring, reducing the number of possibilities we need to explore.

Next, we note that since parallel wiring reduces resistance, it is not worth including small resistors, when maximalizing resistance. Thus, if two identical resistors have less resistance than the third one, it can be assumed that the parallel connection of two identical resistors and the third in series with them will have the second largest resistance.

In addition, we can note that if one branch of the parallel wiring has significantly more resistance than the other, the current will mostly flow through the branch with lesser resistance, which puts up little resistance. In the limiting case, when the resistance of this branch goes to zero, current can flow through the circuit with virtually no resistance and no current flows through the other branch. On this basis, it can be concluded that if two equal resistors have more resistance than the third one, a series connection of a pair of identical resistors in one branch and to them in parallel a smaller resistance will have the second smallest resistance.

This reduced the number of cases we have to examine to four:

- If the smallest resistance is missing, the largest resistance corresponds to the series circuit of all three resistors. Then, the missing resistance corresponds to the parallel circuit of all three resistors.
  - If the same resistors have lower resistance than the third, the two identical resistors in parallel and the third in series corresponds to the second largest resistance.
  - If the same resistors have resistance greater than the third, the two identical resistors in series in one branch and a third in parallel to them corresponds to the smallest known resistance[3].
- In the absence of the largest resistance, the smallest resistance corresponds to the parallel wiring of all three resistors. Then the missing resistance corresponds to the three resistors in series.
  - If the same resistors have smaller resistance than the third, two identical resistors in parallel and the third in series corresponds to the largest known resistance[4].
  - If the identical resistors have greater resistance than the third, two identical resistors in series on one branch and the third in parallel to them corresponds to the second smallest resistance.

Let us explore these possibilities. Doing so, we may notice that the largest resistance on the list is significantly larger than the next two resistances on the list. Therefore, we may assume that the largest resistance corresponds to the serial connection of all three resistors, and that the next two resistances correspond to the circuits in which one resistor is serially connected to a couple of two resistors conected in parallel. Thus, let us start with the first two options.

We already know the resistance of all three resistors in series. The resistance of identical resistors R_1 in parallel and R_2 in series has a resistance $\frac{R_1}{2} + R_2$. Solving this system of two equations gives the resistances $R_1 = 70\ \Omega$ and $R_2 = 105\ \Omega$. We can easily verify that the remaining resistances from the problem correspond to one of the circuits with such resistors. We do not need to investigate other possibilities – we have already found the solution.

The missing resistance corresponds to all three resistors in parallel, i.e. 26,25 Ω.

22 First we have to decide which strategy to use when covering the window with foil. Apparently, we will be able to cover the entire window twice and still have some foil left over. Using the remaining foil, we will

[3] The resistance of a parallel connection of all three resistors, which certainly has a smaller resistance, is unknown

[4] Resistance of series wiring of all three resistors, which certainly has the greater resistance, is unknown

achieve the greatest filtering effect if we cover that area of the window surface which has the smallest number of layers covering it so far. That means, we will try to cover the entire window as uniformly as possible.

The surface of the foil is 1,5 m^2, the surface of the window is $S = 0{,}64$ m^2. To cover the entire window twice, we will use 1,28 m^2 of foil and will have 0,22 m^2 $=: S_3$ of foil left over. That means that the surface S_3 will be covered three times and the surface $S_2 = S - S_3 = 0{,}42$ m^2 will be covered twice.

How much power will pass through multiple layers of foil can be easily calculated. Each layer will allow 80 % of incident light to pass through. It then follows that two layers will allow $\alpha_2 = 80\ \% \cdot 80\ \% = 64\ \%$ and three layers of foil will allow $\alpha_3 = 80\ \% \cdot 64\ \% = 51{,}2\ \%$ of light to pass through.

Knowing the area covered by two and three layers respectively, the total power passing through the entire window can be calculated as

$$P' = (\alpha_2 S_2 + \alpha_3 S_3)P = \left(0{,}64 \cdot 0{,}42\ \mathrm{m}^2 + 0{,}512 \cdot 0{,}22\ \mathrm{m}^2\right) \cdot 1366\ \mathrm{W/m}^2 \doteq 521\ \mathrm{W}. \tag{22.1}$$

23 Let Lucy have zero potential energy and some kinetic energy at the moment she leaves the ground. At the highest point of the jump, at height h, she stops momentarily, so she only has potential energy mgh, where m is Lucy's mass. This means that her kinetic energy at the start of the jump is equal to mgh, because the law of conservation of mechanical energy holds during the jump.

If Lucy wants to jump off the asteroid, she has to reach the infinity. Here, we can no longer use the relation for the potential energy in a homogeneous gravitational field as we did for the jump on the Earth. Instead, we have to take into account the radial field of the asteroid. In the limiting case, when the asteroid has the largest possible mass for Lucy to jump off, Lucy arrives at infinity and stops there after an infinitely long time.

Lucy's potential energy at the surface of the asteroid is $-\frac{GMm}{R}$, where M is the mass of the asteroid and R is its radius. At the start of the jump she gains kinetic energy equal to mgh. At infinity, Lucy stops moving and her potential energy is zero. Due to the conservation of energy, the sum of the kinetic and potential energy at the moment of lift-off is equal to the sum of the kinetic and potential energy at infinity,

$$mgh - \frac{GMm}{R} = 0 + 0. \tag{23.1}$$

From here we can express the mass of the asteroid in the limiting case,

$$M = \frac{Rgh}{G}, \tag{23.2}$$

and then the density as

$$\rho = \frac{M}{V} = \frac{3gh}{4\pi R^2 G} \approx 5618\ \mathrm{kg/m}^3.$$

24 Let us denote the mass of the Sun as $M_\odot$ and the mass of the Earth as $M_\oplus$. Their mutual distance is R. Both of these objects revolve around the common centre of mass with some angular velocity. First, we will need to find the centre of mass. Let us denote the distance between the Sun and the centre of mass as r. Then

$$M_\odot r = M_\oplus (R - r) \tag{24.1}$$

which yields

$$r = \frac{M_\oplus}{M_\odot + M_\oplus} R. \tag{24.2}$$

The Sun revolves around the centre of mass on a circular orbit with radius r and speed v. After realising that this is caused by the gravitational force we get

$$M_\odot \frac{v^2}{r} = G \frac{M_\odot M_\oplus}{R^2}. \tag{24.3}$$

Now it is enough to plug in the radius r from the equation 24.2 and to express v. We obtain

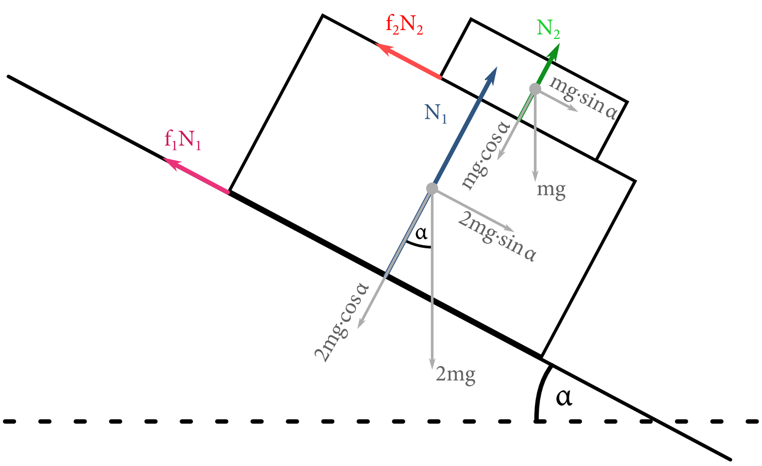
$$v = \sqrt{G \frac{M_\oplus^2}{R(M_\odot + M_\oplus)}}, \tag{24.4}$$

which after evaluation yields $v \approx 9$ cm/s.

When the Sun moves perpendicular to the line connecting the barycentre and the alien observer, its radial velocity is zero. If it moves towards of away from the observer, its radial velocity is $\pm v$. Therefore v is exactly the amplitude of the radial velocity of the Sun.

25 In plotting the forces acting on the plate and the schnitzel, we come across the main non-trivial idea of the problem. What is the relative acceleration of the plate and the schnitzel? In fact, the schnitzel has a greater or equal acceleration than the plate, a reason we will now discuss. Consider the situation where f_1, the coefficient of shear friction between the plate and the table, is zero. In that case, the plate and the schnitzel will move with acceleration $g \sin \alpha$. Thus, their relative acceleration will be zero and the schnitzel will not move relative to the plate.

Let's put this situation in the horizontal direction. Nothing will move. If we add a force to represent the frictional shear force between the plate and the table (F_t), that is the force that acts only on the plate, we can easily see that the schnitzel will want to stay in place relative to the table. Therefore, the plate (T) will exert a frictional force on the schnitzel, which will prevent it from staying still. T will cause no more acceleration than F_t causes. We can see that the schnitzel will go down faster than the plate.



25.1. Ábra: *Sketch of the forces acting on the plate and the schnitzel.*

From the image we can see all the forces and we can write the equations of motion for the plate,

$$2ma = 2mg\sin\alpha - (2m + m)g f_1 \cos\alpha + mg f_2 \cos\alpha, \tag{25.1}$$

where f_2 is the coefficient of shear friction between the plate and the schnitzel.

The first term on the right-hand side of the equation 25.1 represents the component of the gravitational force that moves the plate down the table, the second represents the shear frictional force between the plate and the table (for a mass of $3m$, because the top of the plate also pushes on the schnitzel) and the third term is the reaction force to the shear friction force between the schnitzel and the plate.

After adding the values and expressing a, we get the resulting acceleration of the plate $a \approx 1{,}08\ \mathrm{m/s^2}$.

26 Jelölje ℓ a keletkező kettős rugó hosszát. Az első rugót ebben az állapotban $k\ell$ erő feszíti, a második, vékonyabb rugó pedig $K(L-\ell)$ erővel van összenyomva. Ez a két erő egyenlő kell, hogy legyen, vagyis

$$k\ell = K(L-\ell) \quad \Rightarrow \quad \ell = \frac{KL}{k+K}. \tag{26.1}$$

Most képzeljük el, hogy a két rugó által alkotott rendszert megnyújtjuk $\Delta\ell$ távolságra, vagyis a teljes hosszúsága $\ell + \Delta\ell$ lesz. Mekkora a most fellépő erő? Az első rugóban $k(\ell+\Delta\ell)$ erő ébred a megnyúlás hatására, a második rugó $K(L-\ell-\Delta\ell)$ erővel van összenyomva. Ez a két erő egymással ellentétes irányú, vagyis ki kell őket vonnunk egymásból. Az eredő erő eszerint

$$F = k(\ell+\Delta\ell) - K(L-\ell-\Delta\ell). \tag{26.2}$$

Végezetül, a 26.1 egyenletet kihasználva az előző egyenletet a

$$F = (K+k)\,\Delta\ell \tag{26.3}$$

alakra tudjuk egyszerűsíteni. Ebből már látható, hogy a rendszer közös merevségi tényezője $k+K$.

27 Obviously, we will use physical quantities given in the problem statement. Let the guitar string frequency be

$$f \sim F^a M^b L^c \text{ for some } a, b, c \in \mathbb{R}. \tag{27.1}$$

We did not write the equal sign because we are not interested in numerical constants. Those are not determinable by dimensional analysis. Values of a, b, c are to be determined so that the product of physical quantities in 27.1 has dimension of frequency, that is $\mathrm{s^{-1}}$. Apart from that, newton expressed in SI base units is $\mathrm{kg\cdot m\cdot s^{-2}}$.

The right side of the 27.1 in the language of dimensions is

$$\left(\mathrm{kg\cdot m\cdot s^{-2}}\right)^a \mathrm{kg}^b \mathrm{m}^c = \mathrm{kg}^{a+b}\mathrm{m}^{a+c}\mathrm{s}^{-2a}, \tag{27.2}$$

which has to be of same dimension as frequency, so

$$\mathrm{s}^{-1} = \mathrm{kg}^{a+b}\mathrm{m}^{a+c}\mathrm{s}^{-2a}. \tag{27.3}$$

In order for this equality to be true, the exponent of each physical quantity has to be same on both sides of the equation. A swift look shows that $a = \frac{1}{2}$, which immediately leads to $b = -\frac{1}{2}$ and $c = -\frac{1}{2}$. Thus, a guitar string vibrates with frequency $f \sim \sqrt{\frac{F}{ML}}$.

Fun fact, if we calculated it properly, the fundamental frequency of a guitar string is $f = \frac{1}{2}\sqrt{\frac{F}{ML}}$.

28 From the problem statement

$$N(m > M) = 10^{a-bM}, \tag{28.1}$$

so for the number of earthquakes in the interval 4,0 – 4,9 we can write

$$\begin{aligned} N(3{,}95 \le m < 4{,}95) &= N(m \ge 3{,}95) - N(m \ge 4{,}95) \\ &= 10^{a-3{,}95b} - 10^{a-4{,}95b} \\ &= 10^a \cdot 10^{-4{,}95b}\left(10^b - 1\right) \\ &= \alpha \cdot \beta^{-4{,}95}(\beta - 1), \end{aligned} \tag{28.2}$$

where we introduced the notation $10^a \equiv \alpha$ and $10^b \equiv \beta$. By analogy, for the interval 5,0 – 5,9 we have

$$\begin{aligned} N(4{,}95 \le m < 5{,}95) &= N(m \ge 4{,}95) - N(m \ge 5{,}95) \\ &= 10^{a-4{,}95b} - 10^{a-5{,}95b} \\ &= 10^a \cdot 10^{-5{,}95b}\left(10^b - 1\right) \\ &= \alpha \cdot \beta^{-5{,}95}(\beta - 1). \end{aligned} \tag{28.3}$$

For the number of earthquakes with magnitude 5,0,

$$\begin{aligned} N(4{,}95 \le m < 5{,}05) &= N(m \ge 4{,}95) - N(m \ge 5{,}05) \\ &= 10^{a-4{,}95b} - 10^{a-5{,}05b} \\ &= 10^a \cdot 10^{-4{,}95b}\left(1 - 10^{-0{,}1b}\right) \\ &= \alpha \cdot \beta^{-4{,}95}\left(1 - \beta^{-0{,}1}\right). \end{aligned} \tag{28.4}$$

When we divide 28.2 by 28.3, we get

$$\beta = \frac{N(3{,}95 \le m < 4{,}95)}{N(4{,}95 \le m < 5{,}95)}. \tag{28.5}$$

Equation 28.2 moreover implies,

$$\alpha \cdot \beta^{-4{,}95} = \frac{N(3{,}95 \le m < 4{,}95)}{\beta - 1}. \tag{28.6}$$

For the number of earthquakes we are looking for, we get

$$N(4{,}95 \le m < 5{,}05) = N(3{,}95 \le m < 4{,}95) \cdot \frac{1-\beta^{-0{,}1}}{\beta - 1}, \tag{28.7}$$

After evaluation with given values from the problem statement $N(4{,}95 \le m < 5{,}05) \approx 170$.

29 The heating station heats the school with its thermal power. We need to find how many joules per second are transferred from the mysterious liquid in the radiators to the air in the building. If the liquid changes its temperature by ΔT, its volume changes by $V_0 \beta \, \Delta T$, where V_0 is its original volume and β is volumetric coefficient of thermal expansion.[5]

The change of temperature is thus

$$\Delta T = \frac{\Delta V}{V_0 \beta}. \tag{29.1}$$

The heat transferred from the liquid to the building is simply

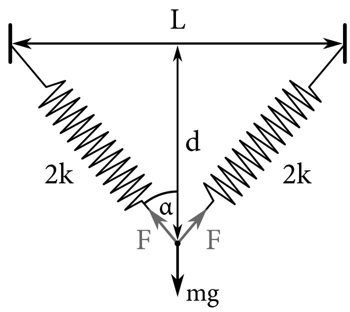
$$Q = mc\,\Delta T, \tag{29.2}$$

where m is mass of the mysterious liquid and c is its specific heat capacity. The liquid has the same density as water, so the outgoing 9,9 l per second weigh $m = 9{,}9$ kg. After plugging this into 29.2, we find out that $Q \approx 2{,}2$ MJ. Since this heat is dissipated in one second, the power of the heat station is 2,2 MW.

30 Let us first note that if Patrick with mass m is suspended from a spring with a stiffness k, then its extension will be

$$\Delta L = \frac{mg}{k}. \tag{30.1}$$

If Patrick is suspended by the centre of the spring as in the problem, he will be suspended by height d, see figure 30.1. We can split the whole spring in half into two springs of stiffness $2k$. The rest length of each is $\frac{L}{2}$ and the actual length of each is $\sqrt{\left(\frac{L}{2}\right)^2 + d^2}$.



30.1. Ábra: *Geometry of a suspended Patrick*

[5] This relationship is in fact exponential, but as the change in temperature is small we can use a linear approximation.

The force F that each of the springs exerts on Patrick is then

$$F = 2k\left(\sqrt{\frac{L^2}{4} + d^2} - \frac{L}{2}\right). \tag{30.2}$$

In order for Patrick to be in equilibrium, the forces from the springs must balance with the gravitational force. Thus, the equation

$$mg = 2F\cos\alpha \tag{30.3}$$

must hold, where we can see from the geometry that

$$\cos\alpha = \frac{d}{\sqrt{\frac{L^2}{4} + d^2}}. \tag{30.4}$$

When we substitute the force F and $\cos\alpha$ to the equations 30.2, 30.3, we get

$$mg = 4k\left(\sqrt{\frac{L^2}{4} + d^2} - \frac{L}{2}\right)\frac{d}{\sqrt{\frac{L^2}{4} + d^2}}. \tag{30.5}$$

We now divide this equation by the stiffness k and use the equation 30.1,

$$\begin{aligned} \Delta L &= 4\left(\sqrt{\frac{L^2}{4} + d^2} - \frac{L}{2}\right)\frac{d}{\sqrt{\frac{L^2}{4}4 + d^2}} \\ -4Ld &= \sqrt{L^2 + 4d^2}(\Delta L - 4d) \end{aligned} \tag{30.6}$$

and we square this to get rid of the square root

$$\left(L^2 + 4d^2\right)(\Delta L - 4d)^2 = 16L^2d^2. \tag{30.7}$$

Now we get a quartic equation

$$L^2\,\Delta L^2 - 8L^2\,\Delta Ld + 4\,\Delta L^2d^2 - 32\,\Delta Ld^3 + 64d^4 = 0 \tag{30.8}$$

with an unknown length d. This one is quite difficult to solve analytically, but after plugging in all numerical constants $L = 1$ m and $\Delta L = 2$ m, we can solve it numerically.

The simplest way to find the root of the equation numerically is by binary search. Let us denote the left-hand side of the equation 30.8 as a function $f(d)$. First, we guess some values for d and evaluate $f(d)$. We get

$$f(0) = 4, \quad f(1/2) = -4 \quad \text{and} \quad f(1) = 4. \tag{30.9}$$

From the signs of these results, it follows that one root will lie in the interval 0 – 0,5 m and the other in the interval 0,5 – 1 m. Next, we proceed by dividing these intervals in half and, according to the sign of the function $f(d)$, narrow the interval each time until we reach the desired precision.

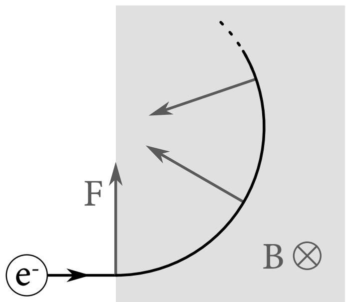
We get two roots, $d = 0{,}2655$ m and $d = 0{,}9416$ m. However, after plugging it back into the equation 30.6, we find, that the first of the solutions does not satisfy the equation. Thus, this solution is not physically correct and was generated in step, where we have made a square from equation 30.6. Thus, the height by which Patrick suspends is $d \approx 0{,}94$ m.

31 Electrically charged particles are subject to both electric and magnetic forces. Whilst electric fields simply act repulsively or attractively, analogous to gravity, magnetic fields have a more complex effect, which is described by the equation

$$\vec{F} = q\vec{v} \times \vec{B}. \tag{31.1}$$

Let us describe the situation in cartesian coordinates. Let us say the electron comes in the positive x direction and the magnetic field everywhere is in the negative z direction (a common convention). Then, at the moment the magnetic field is turned on, the force on the electron acts in the positive y direction. As the trajectory of the electron curves towards the positive y direction, the direction of the force rotates simultaneously so that it is always perpendicular to both the instantaneous velocity and the magnetic field.

As the z-component of the electron's velocity is initially zero and the z-component of the force must be always zero, the electron is trapped in the xy plane. And since the force is always perpendicular to the electron's velocity, the velocity's magnitude will stay constant, only its direction changes, and that at a constant rate (since no quantity in the force law changes magnitude). Therefore, the electron moves in a circle.



31.1. Ábra: *The electron's trajectory*

Since the direction of the motion is always perpendicular to the magnetic field, we can express the magnitude of F by changing the cross product in 31.1 to a simple multiplication of v and B. Since this is the force causing the circular motion, we can equate it with the expression for centripetal acceleration and isolate R,

$$\begin{aligned} m_e\frac{v^2}{R} &= q_e v B, \\ R &= \frac{m_e v}{q_e B}. \end{aligned} \tag{31.2}$$

This is called Larmor radius. We are interested in the time it takes the electron to traverse a quarter of the orbit's circumference, with this segment having the length of $\frac{\pi}{2}R$. This time is then

$$t = \frac{\pi}{2v}R = \frac{\pi}{2v}\frac{m_e v}{q_e B} = \frac{\pi m_e}{2q_e B}. \tag{31.3}$$

We know all the quantities in this formula – elementary charge, electron mass, strength of the magnetic field – hence we obtain $t \approx 1$ ns.

32 If we denote the mass of the balloon as M, then $F = V\rho g - Mg$, where ρ is the density of water. Similarly, the force that the balloon eventually exerts on the bottom is $F' = Mg - V'\rho g$, since water is incompressible and, therefore, does not change its density. The problem states that the compression of the balloon is isothermal, so $pV = p'V'$. It remains for us to find the values of the pressure. Initially the balloon is at atmospheric pressure $p = p_{\text{atm}}$, since the water pressure acts on the balloon in the same way as on the piston. At the bottom, however, the pressure is increased due to the loaded piston mg/S, and secondly by the column of water above the balloon. So for V' we get

$$\begin{aligned} p_{\text{atm}} V &= p'V' = (p_{\text{atm}} + mg/S + h\rho g)V' \\ V' &= \frac{p_{\text{atm}}}{p_{\text{atm}} + mg/S + h\rho g} V \end{aligned} \tag{32.1}$$

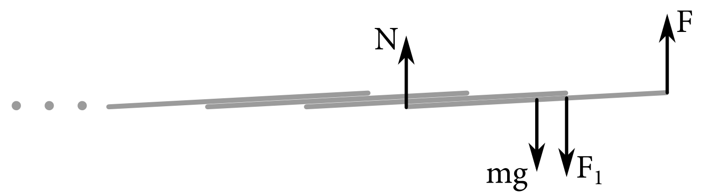
This result has to be substituted into the relation for F' to get

$$F' = Mg - V'\rho g = (V - V')\rho g - F = \frac{V\rho g}{1 + \frac{p_{\text{atm}}/g}{m/S + h\rho}} - F \approx 0{,}126 \text{ N}. \tag{32.2}$$

33 The card at the bottom is our first card. If we want to lift it up a tiny bit, the total torque acting on that card must be zero with respect to the axis of rotation. That is in our case the edge of the card that is below all the other cards,

$$F\ell = \frac{1}{2} mg\ell + (\ell - \Delta\ell)F_1, \tag{33.1}$$

where F is the force that Justine exerts on the card and F_1 is the force that the second card (and all the other cards through it) exerts on the first card.



33.1. Ábra: *Forces acting on cards*

For an infinite number of cards, we can assume that $F = F_1$. If, in fact, Justine removes the first card, there are still infinitely many cards she needs to lift up. With this assumption, we use the equation 33.1 to get

$$F = \frac{mg\ell}{2\,\Delta\ell}. \tag{33.2}$$

34 We shall consider the situation first in the non-inertial frame of reference with the observer in the axis of rotation in the roundabout's centre, rotating with the roundabout. In this frame of reference, the roundabout is motionless. There is a fictitious centrifugal force acting on Snow White given by $F_o = m\omega^2 r$

which she has to fight against to climb to the centre. Since Snow White does work, the total energy of the system is not conserved. However, since this force is purely radial, it has zero torque.

As Snow White starts moving, there is a second fictitious force at play: Coriolis force. This has the magnitude of $F_c = 2m\omega v_\perp$, where $v_\perp$ is the component of Snow White's velocity perpendicular to the axis of rotation. This force is perpendicular to both the axis of rotation and $v_\perp$, so it has nonzero torque. As Snow White climbs to the centre, she exerts a torque on the roundabout, transferring her angular momentum to the seven dwarves. As we've shown, the energy of the system isn't conserved, but the angular momentum is – this is clear when we look at it in an inertial frame of reference, since there is only one external force acting on the roundabout, and that is the force the ground exerts on the axle to hold it in place, which clearly has zero torque.

Since Snow White's angular momentum will be zero at the end of the manoeuvre, by equating the initial and final angular momentum we obtain

$$(m_s + 7m_t)\omega_1 r^2 = 7m_t\omega_2 r^2, \tag{34.1}$$

where m_s is Snow White's mass, m_t$ is the mass of one dwarf, ω_1 is the initial angular velocity and ω_2 is the final angular velocity.

We are given that $\omega_1 = \frac{2}{3}\omega_2$, which yields $(m_s + 7m_t) = \frac{3}{2} \cdot 7m_t$. This gives us the mass of one dwarf,

$$m_t = \frac{2}{7}m_s = 16\ \text{kg}. \tag{34.2}$$

35 Let us displace the disc from its equilibrium position by a distance Δx. The pressure in the compressed chamber of the cylinder before its base gets torn off will then increase by $\Delta p(\Delta x)$ and in the other chamber pressure drops by the same amount[6]. Therefore a force with magnitude $F_1(\Delta x) = 2\,\Delta p(\Delta x)S$ acts on the disc in an attempt to move the disc back into its equilibrium position (in the middle of the cylinder). In the small displacement approximation, the function $\Delta p(\Delta x)$ is linear.

In the cylinder with its base missing, the displacement of the disc by Δx from its equilibrium (which is still in the middle of the cylinder) causes the same difference in pressure $\Delta p(\Delta x)$ in the undisturbed chamber, but no difference in the other chamber. The force acting on the disc is now only

$$F_2(\Delta x) = \Delta p(\Delta x)S = \frac{1}{2}F_1(\Delta x). \tag{35.1}$$

This setup is in the first case equivalent to the disc being connected to a spring with appropriate stiffness k_1 and the other case is equivalent to the disc being connected to a spring with stiffness $k_2 = \frac{1}{2}k_1$. The period of small oscillations of a mass on a spring is $T = 2\pi\sqrt{\frac{m}{k}}$ with m being the mass of the disc and k being the spring stiffness, which means that the unknown ratio of the periods is

$$q = \frac{2\pi\sqrt{\frac{m}{k_1}}}{2\pi\sqrt{\frac{m}{k_2}}} = \sqrt{\frac{k_2}{k_1}} = \frac{\sqrt{2}}{2}. \tag{35.2}$$

[6] All of this is true only in the case of small displacements, because then the pressure as a function of displacement can be linearised.

36 How does the wheel accelerate after falling on the ground? Well, if the wheel has mass m, it's pressed onto the ground by gravitational force of size mg. Also, to the floor it seems as if the wheel is moving – since it's rotating with angular velocity ω_0, at the point of contact its surface moves at cross-radial velocity of $R\omega_0$ relative to both the wheel's centre of mass and the ground, where $R = 2{,}5$ m is the wheel's radius. This means there will be friction acting on the wheel at the point of contact with magnitude fmg (where f is the coefficient of friction) in the direction parallel to the ground and opposing the wheel's rotation. The wheel's centre of mass will begin linearly accelerating along the ground with its transverse velocity given by $v_x(t) = at = fgt$, but the friction also exerts torque that linearly decelerates the wheel's rotation, so that its angular velocity is given by

$$\omega(t) = \omega_0 - \frac{fmgR}{I}t, \tag{36.1}$$

where I is the moment of inertia of the wheel.

Friction, of course, doesn't act indefinitely – it will cease the moment the wheel stops slipping. This occurs once the cross-radial velocity at the surface $R\omega(t)$ equals the translational velocity $v_x(t)$, and the wheel reaches a stable rolling motion. Let's find the time t_v at which this occurs:

$$R\left(\omega_0 - \frac{fmgR}{I}t_v\right) = fgt_v \quad \Rightarrow \quad t_v = \frac{R\omega_0}{fg\left(1 + \frac{mR^2}{I}\right)}. \tag{36.2}$$

The final translational velocity of the wheel is then simply

$$v_v = v_x(t_v) = fgt_v = \frac{R\omega_0}{1 + \frac{mR^2}{I}}. \tag{36.3}$$

This is a known variable: what we want to find is the hamster's running speed before the breakage v_s, which is simply equal to the initial cross-radial velocity $R\omega_0$,

$$v_s = R\omega_0 = v_v\left(1 + \frac{mR^2}{I}\right). \tag{36.4}$$

We see that we still need to find the mass and the moment of inertia of the wheel. These can be found by linearly superimposing the properties of each metallic component. The moments of inertia of the 50 small rods on the circumference and circular hoops are trivial to calculate: they all lie at the same distance from the axis of rotation, and even if we cut them up into tiny parts, each part is at distance R from the axis of rotation. Therefore, their total moment of inertia is $50\lambda AR^2 + 2\lambda(2\pi R)R^2$, where A is the length of each small rod.

The two diametral rods are more tricky. Each has mass $2R\lambda$, and in literature we can find that the moment of inertia of a rod of length X and mass M rotating about an axis passing through its centre and perpendicular to the rod itself is $\frac{1}{12}MX^2$.

We have $X = 2R$ and by substituting the mass we find that each diametral rod has moment of inertia of $\frac{2}{3}\lambda R^3$. Hence the total moment of inertia of the wheel is

$$I = \lambda R^2\left(50A + \left(4\pi + \frac{4}{3}\right)R\right). \tag{36.5}$$

We swiftly express the wheel's mass from its dimensions and linear density as

$$m = \lambda(50A + (4\pi + 4)R), \tag{36.6}$$

hence the hamster's running speed is equal to

$$v_s = v_v\left(1 + \frac{50A + (4\pi + 4)R}{50A + \left(4\pi + \frac{4}{3}\right)R}\right) \doteq 2{,}11\ \mathrm{m/s}. \tag{36.7}$$

37 At first glance, it may seem that the satellite would leave the Sun's gravity field. However, if the force is sufficiently small, the satellite will still be bound to the Sun. Its trajectory will not be a conic section anymore but it will remain on some curve which will have a point with maximal radial distance from the Sun. We are looking for force F so that the maximum will be in radial distance of $2R$ from the Sun.

Before the engines started firing, the satellite was orbiting the Sun on a circular trajectory with radius R with speed of $v_0 = \sqrt{\frac{GM}{R}}$. After the engines started firing, the trajectory changed but the law of conservation of angular momentum still holds since the force F has only radial component. Let us denote v the speed of the satellite in distance $2R$. As this is the furthermost point from the Sun on the trajectory, the line joining the Sun and the satellite is perpendicular to its velocity. Therefore, we can write

$$v_0 R = v 2R \tag{37.1}$$

and thus

$$v = \frac{v_0}{2} = \frac{1}{2}\sqrt{\frac{GM}{R}}. \tag{37.2}$$

Next, we use the law of conservation of energy. We must be careful, since the force F does work, too. We divide the trajectory into many small pieces $\Delta\vec{s}$ and calculate the work done $\Delta W = \vec{F}\cdot\Delta\vec{s}$ by the force on the piece, and then sum all ΔW. If we decompose $\Delta\vec{s}$ into radial and transversal component, the dot product can be calculated as product of magnitude of the force $\vec{F}$ and magnitude of $\Delta\vec{s}$ as a result of properties of dot product. Therefore, the total work done by the engines from the start of their firing to the moment when the satellite reaches distance $2R$ from the Sun is equal to FR (the heliocentric distance of the satellite changes by R). Thusly, the law of conservation of energy can be written as

$$\frac{1}{2}mv_0^2 - G\frac{Mm}{R} = \frac{1}{2}mv^2 - G\frac{Mm}{2R} - FR. \tag{37.3}$$

Initial velocity is $v_0 = \sqrt{\frac{GM}{R}}$ and velocity in the distance $2R$ is expressed in 37.2. Now we simply solve for F. The result is

$$F = \frac{1}{8}\frac{GMm}{R^2}. \tag{37.4}$$

38 We need to realise what forces act on the golden sheet. Firstly, we have the gravitational force given by the Newton's gravitational law. Secondly, the reflected photons transfer momentum to the sheet. We use the alternate form of the Newton's second law $F = \frac{\Delta p}{\Delta t}$, where Δp is the difference in momentum. The difference in momentum of the photons that are reflected during a time Δt from the sheet is $\Delta p = \frac{2E}{c}$, where E is the energy incident on the sheet. The photons hit the sheet perpendicularly[7] and after reflection, they have an equal momentum in the opposite direction. Therefore the difference in momentum is doubled.

If the star radiates some energy E_c, the sheet, which is located in distance D, reflects

$$E = \frac{A}{4\pi D^2} E_c, \tag{38.1}$$

where A is the unknown area of the sheet.

The radiated energy can be determined from the Stefan-Boltzmann law for black body radiation as

$$E_c = P\,\Delta t = \sigma T^4 S\,\Delta t = \sigma T^4 4\pi R^2\,\Delta t. \tag{38.2}$$

The force exerted by the radiation must be equal to the gravitational force. This yields the equation

$$\frac{GMm}{D^2} = \frac{\Delta p}{\Delta t} = \frac{2E}{c\,\Delta t} = \frac{2A\sigma T^4 4\pi R^2}{4\pi D^2 c}, \tag{38.3}$$

from where

$$A = \frac{GMmc}{2\sigma T^4 R^2} \tag{38.4}$$

Noticeably, the area density does not depend on the distance from the star. This is an expected result, because both forces are proportional to D^{-2}.

39 This problem, as is usual in special relativity, can be approached in two ways: rigorously or with a trick. Let us denote the flight time of the journey's duration as $\tau = 2\,600\,000$ years and the distance between galaxy M31 and the Earth as $D = c\tau = 2\,600\,000$ light-years.

Rigorous approach

Let us consider the situation in two reference frames, writing down the spacetime coordinates of the Earth and galaxy M31 in each. In the Earth's frame of reference let the Earth be at $t = 0, x = 0$. Then, galaxy M31 is at $t = t_G, x = D$, where D is the given distance and $t_G = \frac{D}{v}$, where v is the traveller's velocity which we want to find. In the traveller's reference frame, these coordinates change under a Lorentz transform. The Earth is still at $t' = 0$, $x' = 0$, but galaxy M31 is now at

$$t' = \gamma\left(t - \frac{vD}{c^2}\right), \quad x' = \gamma(D - vt), \tag{39.1}$$

where $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$ is the Lorentz factor.

Since in this second reference frame the traveller is stationary, the difference of the time coordinates of the Earth and galaxy M31 (equal to t') is also the proper (and flight) time which the traveller measures as the duration of the journey – hence $t' = \tau$. Let us substitute the derived expression for t and express the traveller's

[7] we know this because $D \gg R$

velocity

$$\tau = \gamma D\left(\frac{1}{v} - \frac{v}{c^2}\right) = \frac{1}{v}\gamma D\left(1 - \frac{v^2}{c^2}\right) = \frac{1}{v}\gamma D\gamma^{-2} = \frac{D}{v\gamma},$$

$$\frac{D}{\tau} = \frac{v}{\sqrt{1-\frac{v^2}{c^2}}} = \sqrt{\frac{c^2}{\frac{c^2}{v^2}-1}}, \tag{39.2}$$

$$v = \frac{c}{\sqrt{1+\frac{c^2\tau^2}{D^2}}}.$$

But since $\frac{\tau}{D}$ is simply the inverse of the speed of light c^{-1} (this is how these quantities were given – we could have specified a different value of τ and the solution would be different), hence

$$v = \frac{c}{\sqrt{2}}. \tag{39.3}$$

Trick approach

Since we are interested in the flight time, which is the traveller's proper time, let us imagine we are aboard the travelling spaceship and consider which relativistic effects we observe. Clocks moving relative to us slow down, but that is not of interest to us. Moreover, the distances between points moving relative to us shorten by the Lorentz factor – and this is the effect that causes us to measure a shorter duration of our journey. So if we move at the velocity v, the observed distance to our destination is $D_0 = D\sqrt{1-\frac{v^2}{c^2}}$ and the journey duration is $\tau = \frac{D_0}{v}$. And since we are given $\tau = \frac{D}{c}$, we find

$$\frac{D\sqrt{1-\frac{v^2}{c^2}}}{v} = \frac{D}{c} \tag{39.4}$$

From this it is trivial to express the expected result we found in 39.3.

40 Let us consider a wall of thickness h, area S, and thermal conductivity λ. If we maintain its faces at temperatures t_1 and t_2, respectively, a heat flow

$$P = \frac{\lambda S}{h}(t_2 - t_1). \tag{40.1}$$

will be present.

To simplify matters, let us define coefficient $\lambda S/h =: X$. Now, let us look at a wall with two layers with coefficients X_1 a X_2, respectively. Continuity demands that

$$P = X_2(t_2 - t) = X_1(t - t_1), \tag{40.2}$$

where t is the temperature of the inter-layer interface. This temperature can be computed as

$$t = \frac{X_1 t_1 + X_2 t_2}{X_1 + X_2}, \tag{40.3}$$

and we can write an equation for the heat flow

$$P = \frac{X_1 X_2}{X_1 + X_2}\left(t_2 - t_1\right). \tag{40.4}$$

We can observe that a composite coefficient can be defined as

$$X_{12} \coloneqq \frac{X_1 X_2}{X_1 + X_2}. \tag{40.5}$$

With these preparations, we are ready to tackle the task itself.

Before styrofoam installation, the heat flow from Bob to neighbour was

$$P = \frac{X_{\text{floor}} X_{\text{ceiling}}}{X_{\text{floor}} + X_{\text{ceiling}}}\left(t_{\text{Bob}} - t_{\text{neighbour}}\right) = X_{\text{floor + ceiling}}\left(t_{\text{Bob}} - t_{\text{neighbour}}\right). \tag{40.6}$$

If we neglect neighbour's personal heat production, this heat flow is balanced with losses going outdoors

$$P = X\left(t_{\text{neighbour}} - t_{\text{outdoors}}\right), \tag{40.7}$$

where X is (for now) unknown coefficient describing heat losses to outdoors. With vital help of two equations above, we can determine it as

$$X = X_{\text{floor + ceiling}} \frac{t_{\text{Bob}} - t_{\text{neighbour}}}{t_{\text{neighbour}} - t_{\text{outdoors}}}. \tag{40.8}$$

After insulation, the coefficient describing heat flow between Bob's place and his neighbour becomes

$$X_{\text{floor + ceiling + styrofoam}} = \frac{X_{\text{floor + ceiling}} X_{\text{styrofoam}}}{X_{\text{floor + ceiling}} + X_{\text{styrofoam}}}. \tag{40.9}$$

Once again, the flow from Bob to neighbour is balanced with the losses, so

$$X_{\text{floor + ceiling + styrofoam}}\left(t_{\text{Bob}} - t'_{\text{neighbour}}\right) = X\left(t'_{\text{neighbour}} - t_{\text{outdoors}}\right) \tag{40.10}$$

Now, we can simply write

$$t'_{\text{neighbour}} = \frac{X t_{\text{outdoors}} + X_{\text{floor + ceiling + styrofoam}} t_{\text{Bob}}}{X + X_{\text{floor + ceiling + styrofoam}}}. \tag{40.11}$$

We can evaluate this result as 0,625 °C.

Válaszok

1 9

2 41 m

3 $\dfrac{5}{27}$

4 0,073 g/cm^3 = 73 kg/m^3

5 3,6 s = 10^{-3} h

6 $\dfrac{\sqrt{26}}{2}$ s $\doteq$ 2,55 s

7 4,89 h = 17 591 s *vagy* 4 óra 53 perc 11 másodperc.

8 134

9 4 : 3

10 7,68 l

11 151,6 m. *Ezen intervallumon belüli eredmények fogadhatók el* 150 – 152 m.

12 2 Ω

13 1,2 m

14 98,3 kg, *elfogadhatók a* 96 – 99 kg *intervallumba eső eredmények.*

15 $\dfrac{\pi}{2}r^2 + r^2 = \left(\dfrac{\pi}{2} + 1\right)r^2$

16 203 l, elfogadható a 0,2 m^3 = 200 l is.

17 $\dfrac{180°}{2n+1}, n \in \mathbb{N}_0$

18 $\sqrt{300}$ m/s ≐ 17,32 m/s. *A* 17,15 – 17,35 m/s *intervallumon belüli eredmények fogadhatók el.*

19 $\frac{5\sqrt{3}}{4}$ s ≐ 2,17 s

20 314 °C

21 26,25 Ω

22 521 W

23 5618 kg/m^3, *Fogadjuk el a megoldásokat, amelyek a következő intervallumon belül esnek:* 5612 – 5727 kg/m^3.

24 9 cm/s

25 Az 1,08 – 1,10 m/s^2 intervallumba eső eredmények elfogadhatónak tekinthetőek.

26 *A hosszúság* $L\frac{K}{k+K}$, *a merevségi tényező* $k+K$ *lesz. Csak mindkét kifejezés helyessége esetén fogadható el a válasz. Ha csak az egyik kifejezés helyes,* ***ne*** *jelezzük, melyik jó.*

27 $\sqrt{\frac{F}{ML}}$

28 170

29 2,2 MW

30 0,94 m

31 1,0 ns

32 0,126 N, *(az első három számjegy jelentős)*

33 $\frac{mg\ell}{2\,\Delta\ell}$

34 16 kg

35 $\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

36 2,11 m/s

37 $\dfrac{GMm}{8R^2}$

38 $\dfrac{GMmc}{2R^2\sigma T^4}$

39 $\dfrac{c}{\sqrt{2}}$

40 0,625 °C